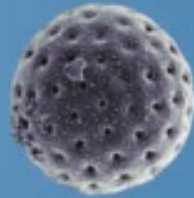


7-2 Scientific Notation

Eye Opener

Forget-me-not flowers from the *Myosotis* genus produce one of the smallest pollen grains. Flowers from the *Mirabilis* genus produce one of the largest pollen grains. How many times as long as the *Myosotis* pollen grain is the *Mirabilis* pollen grain? How do you know?

Mirabilis

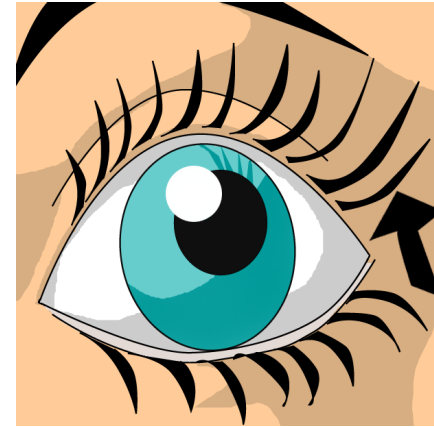


|→ 0.00025 m ←|

Myosotis



|← 0.000002 m →|



Essential Understanding

You can use powers of 10 to write and compare very large or very small numbers easily.

Scientific Notation is a shorthand way to write numbers using powers of 10.





What is scientific notation?

Scientific Notation:

Scientists had a need for a shorter method to express very large and very small numbers. The method they developed is called **scientific notation**. Scientific Notation is built on powers of the base number 10. It was developed before calculators could handle really large and really small numbers.

In scientific notation, every number is written as a product of two factors, the "base", which is always a power of 10 and the "coefficient", which is always a decimal between 1-10.

To write a number in scientific notation:

- To find the coefficient, put the decimal after the first digit and drop all unnecessary zeroes.
- The base is always 10
- To find the exponent of 10, count the number of places from the decimal to the end of the number.

The number 154,000,000,000 is written in scientific notation as

The diagram shows the scientific notation 1.54×10^{11} enclosed in a dark blue rectangular box. Three white arrows point from the box to labels: one points from the '1.54' to the label 'coefficient', one points from the '10' to the label 'base', and one points from the '11' to the label 'exponent'.

The first number (1.54) is called the coefficient. The coefficient must be greater than or equal to 1 and less than 10.

The second number (10) is called the base. It must always be 10. The base number 10 is always written in exponent form.

The third number (¹¹) is called the exponent. It is also referred to as the power of -

The number 123,000,000,000 in scientific notation is written:

$$1.23 \times 10^{11}$$

- *Move the decimal after the 1. Drop all zeroes after the 3.*
- *1.23, the coefficient, is between 1-10*
- *The base is always 10*
- *In 1.23,000,000,000 there are 11 places from the decimal to the end of the number*
- *In counting from the decimal to the end of the number you counted to the right, so the exponent is positive (+11)*

More samples: $3,460 = 3.46 * 10^3$ $6.11 \times 10^5 = 611,000$

172 000 000 000

Write this number in scientific notation by following the steps below.

Step 1: Put the decimal after the first digit.

1.720000000000

Step 2: Find the exponent by counting the number of places from the decimal to the end of the number.

1.720000000000
1 2 3 4 5 6 7 8 9 10 11

Step 3: Drop the zeros to get the coefficient. Multiply it by the base (which is always 10) and include the exponent that you found in step 2.

1.72×10^{11}

Write the following numbers in scientific notation using the steps provided on page 5.

195 000 000 000



Step 1:

1.95

Step 2:

10^{11}

Step 3:

1.95×10^{11}

675 000 000



Step 1:

6.75

Step 2:

10^8

Step 3:

6.75×10^8

56 000 000

Step 1:

5.6

Step 2:

10^7

Step 3:

5.6×10^7

72 000

Step 1:

7.2

Step 2:

10^4

Step 3:

7.2×10^4

Complete the following table.

Number	Coefficient	Base	Exponent
123 000 000 ^	1.23	10	8
98 000 ^	9.8	10	4
785 000 000 000 000	7.85	10	14
670,000	6.7	10	5
946,000,000	9.46	10	8

Fill in the blanks.

$$\boxed{1830000} = \boxed{} \times 10^{\boxed{}}$$

$$\boxed{357000} = \boxed{} \times 10^{\boxed{}}$$

$$\boxed{47100} = \boxed{} \times 10^{\boxed{}}$$

$$\boxed{874000000} = \boxed{} \times 10^{\boxed{}}$$

$$\boxed{} = \boxed{7.89} \times 10^{\boxed{7}}$$

$$\boxed{} = \boxed{9.235} \times 10^{\boxed{9}}$$



What about numbers that are less than one? Can they be written in scientific notation?

Numbers that are less than 1 have a negative exponent. A millionth of a second would look like this:

$$0.000001 \text{ s} = 1.0 \times 10^{-6}$$

Write the following numbers in scientific notation. Remember that the coefficient only has one number before the decimal place.

$$0.00000007 = 7.0 \times 10^{-8}$$

$$0.00056 = 5.6 \times 10^{-4}$$

$$0.0000743 = 7.43 \times 10^{-5}$$

$$0.092 = 9.2 \times 10^{-2}$$

The number .00000436 in scientific notation is written:

$$4.36 \times 10^{-6}$$

- *Move the decimal after the 4. Drop all zeroes before the 4.*
- *4.36 is a number between 1-10*
- *The base is 10*
- *In .000004.36 there are 6 places from the decimal to the end of the number*
- *In counting from the decimal to the end of the number, you counted to the left, so the exponent is negative (- 6).*

More samples: $5.43 \times 10^{-8} = .0000000543$

$$.00023 = 2.3 \times 10^{-4}$$

How the Graphing Calculator Handles Scientific Notation

The graphing calculator uses "E" to indicate scientific notation:

$$1.23\text{E}+11 = 1.23 \times 10^{11} = 123,000,000,000$$

$$.000000000056 = 5.6 \times 10^{-11} = 5.6 \text{ E}-11$$

Write each number in scientific notation:

$$56,900,000 =$$

$$.00985 =$$

$$123,000,000,000 =$$

$$.00000008756 =$$

$$52 \times 10^4 =$$

$$.04 \times 10^{-5} =$$

Convert each number from scientific notation to standard form:

$$3.2 \times 10^{12} =$$

$$2 \times 10^{-11} =$$

$$1.6749 \times 10^{23} =$$

$$3.7 \times 10^{-15} =$$

Multiplying Terms with Exponents:

$$x^3 * x^5 = (x * x * x) * (x * x * x * x * x) = x^8$$

$$x^2 * x^4 = (x * x) * (x * x * x * x) = x^6$$

$$2^3 * 2^5 = 2^8$$

RULE #4:

$$x^a * x^b = \underline{x^{a+b}}$$

When multiplying terms with exponents, if your bases are the same you add your exponents

Apply Rule #4: $x^a * x^b = x^{a+b}$

$$x^7 * x^{23} = x^{30}$$

$$x^5 * x^{-5} = x^0 = 1$$

$$x^{-10} * x^7 = \frac{1}{x^3}$$

$$x^{-4} * x^{-3} = x^{-7} = \frac{1}{x^7}$$

Multiplication in scientific notation

$$(2.1 \times 10^8) (3.2 \times 10^4)$$

Multiply these two numbers together by following the steps below.



Step 1: Multiply the coefficients together.

$$2.1 \times 3.2 = 6.72$$

Step 2: Add the exponents together. (Base 10 always remains the same.)

$$8 + 4 = 12$$

Step 3: Write the product.

$$(2.1 \times 10^8) (3.2 \times 10^4) = 6.72 \times 10^{12}$$

Multiply the following using the steps provided on page 9.

$$(2 \times 10^3)(4 \times 10^4) =$$

Step 1:

Step 2:

Step 3:

$$(5.5 \times 10^7)(1.1 \times 10^4) =$$

Step 1:

Step 2:

Step 3:

$$(3 \times 10^5)(2 \times 10^6) =$$

Step 1:

Step 2:

Step 3:

$$(2.6 \times 10^9)(3.1 \times 10^5) =$$

Step 1:

Step 2:

Step 3:



What happens if the coefficient is more than 10?

$$\text{Example: } (6 \times 10^3) (4 \times 10^4) = \underline{24} \times 10^7$$

The product value is not correctly written in scientific notation because the coefficient is not between 1 and 10. To make it correct, move the decimal point over to the left until the coefficient is between 1 and 10. For each place you move the decimal to the left, the exponent will be increased by one power of ten.

$$(6 \times 10^3) (4 \times 10^4) = 24 \times 10^7 = 2.4 \times 10^8$$

Multiply the following and put the product in proper scientific notation.

$$(9.2 \times 10^4) (3.1 \times 10^5) =$$

$$(4.5 \times 10^8) (4.2 \times 10^5) =$$

$$(7.11 \times 10^4) (3 \times 10^9) =$$

$$(8.3 \times 10^2) (7.2 \times 10^6) =$$



How do you do multiplication of scientific notation if the exponents are negative?

When multiplying numbers in scientific notation that have negative exponents, use the same steps as before but use the rules of addition of signed numbers.

$$\text{Example: } (2 \times 10^{-3})(3 \times 10^{-3}) = 6 \times 10^{-6}$$

Multiply the following numbers:

$$\begin{array}{l} \cdot 000003 \times \cdot 0002 = \cdot 0000000006 \\ (3 \times 10^{-6})(2 \times 10^{-4}) = \end{array}$$

$$(3 \times 2)(10^{-6} \times 10^{-4}) = 6 \times 10^{-10}$$

$$(5 \times 10^{-5})(1 \times 10^{-4}) = 5 \times 10^{-9}$$

$$(5.5 \times 10^{-7})(4.2 \times 10^4) =$$

Now Try these:

$$(2 \times 10^3)(4 \times 10^4) = 8 \times 10^7$$

$$(6 \times 10^5)(7 \times 10^6) = 42 \times 10^{11} = 4.2 \times 10^{12}$$

$$(5.5 \times 10^7)(4.2 \times 10^4) =$$

$$(3 \times 10^{-3})(3 \times 10^{-3}) = 9 \times 10^{-6}$$

$$(2 \times 10^{-3})(3 \times 10^8) = 6 \times 10^5$$

$$(5 \times 10^7)(8 \times 10^{-6})(4.2 \times 10^4) = 1.68 \times 10^7$$

$$40 \times 4.2 = 1.68 \times 10^2$$

Rules for Division in Scientific Notation:

1) Divide the coefficients

2) Subtract the exponents (base 10 remains)

Example 1: $\frac{(6 \times 10^6)}{(2 \times 10^3)} = 3 \times 10^3$

Example 2: $\frac{(2 \times 10^7)}{(8 \times 10^3)} = 0.25 \times 10^4 = 2.5 \times 10^{-1} \times 10^4$
 $0.25 \times 10^4 = 2.5 \times 10^3 = 2.5 \times 10^3$

Now Try these:

$$\frac{9 \times 10^{-6}}{3 \times 10^{-3}} = 3 \times 10^{-3}$$

$$\frac{2 \times 10^3}{4 \times 10^{-8}} = .5 \times 10^{11} = 5 \times 10^{10}$$

$$\frac{3 \times 10^7}{8 \times 10^4} = .375 \times 10^3 = 3.75 \times 10^2$$

$$\frac{3 \times 10^7}{8 \times 10^{-6}} =$$

$$\frac{9 \times 10^8}{2 \times 10^6} =$$

$$\frac{4 \times 10^6}{2 \times 10^4} =$$

$$\frac{3 \times 10^{-6}}{2 \times 10^{-4}} =$$

$$\frac{7 \times 10^{-5}}{2 \times 10^{10}} =$$

$$\frac{1.5 \times 10^{-7}}{3.0 \times 10^4} =$$

Converting powers of ten to decimals and fractions

$$10^{-3} =$$

$$10^4 =$$

$$10^{-5} * 10^3 =$$

$$\frac{10^{-4}}{10^6} =$$

$$10^9 * 10^{-5} =$$

$$\frac{10^{-3}}{10^2} =$$

Converting decimals and fractions to powers of ten

$$\frac{1}{1,000} = 10^{-3} \quad \leftarrow \quad \frac{1}{1000} = \frac{1}{10^3}$$

$$1,000,000 = 10^6$$

$$1,000 * 10,000 = 10^7$$

$$10^3 \cdot 10^4$$

$$\frac{10,000}{100,000} = \frac{10^4}{10^5} = 10^{-1}$$

$$1/1000 * 10/10000 = 10^{-6}$$

$$10^{-3} \cdot \frac{10^1}{10^4} = 10^{-3} \cdot 10^{-3}$$

$$\frac{100/1000000}{1000/10000} =$$

3.5
7.4

$$\begin{aligned} & (2.6 \times 10^3) \\ & (3.5 \times 10^{-7}) \\ & (2.6 \cdot 3.5) \cdot (\underline{10^3} \cdot \underline{10^{-7}}) \\ & 9.10 \cdot 10^{-4} \end{aligned}$$

$$(2.5 \times 10^{-8})(6.4 \times 10^{15})$$
$$(2.5 \times 6.4)(10^{-8} \times 10^{15})$$

$$\boxed{16} \times 10^7$$

$$1.6 \times 10^1 \times 10^7$$

$$\boxed{1.6 \times 10^8}$$