x^0 x^{-1}

7-1: Zero & Negative Exponents

Eye Opener

Copy and complete the table. Make a conjecture about how the value of an exponential expression (an expression containing an exponent) changes when you decrease the exponent by 1. What do you think the value of 5^{-2} is? Explain your reasoning.

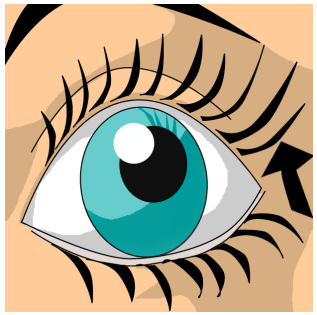
$\frac{1}{25}$

$*[A]^{-1} \Leftarrow \Rightarrow \div[A] = *\frac{1}{[A]}$

÷2

2^x	10^x
$2^4 = 16$	$10^4 = 10{,}000$
$2^3 = 8$	$10^3 = 1{,}000$
$2^2 = 4$	$10^2 = 100$
$2^1 = 2$	$10^1 = 10$
$2^0 = 1$	$10^0 = 1$
$2^{-1} = \frac{1}{2}$	$10^{-1} = \frac{1}{10}$
$2^{-2} = \frac{1}{4}$	$10^{-2} = \frac{1}{100}$

÷ 10



Essential Understanding

In this unit you will learn to extend your understanding of exponents to include zero and negative integer values.

Dividing with Exponents

$x^5 \div x^3 = \frac{x^5}{x^3} = \frac{x*x*x*x*x}{x*x*x} = x^2$

$y^8 \div y^5 = \frac{y^8}{y^5} = \frac{y*y*y*y*y*y*y*y}{y*y*y*y*y} = y^3$

Rule #1:

$x^a \div x^b = x^{a-b}$

When dividing terms with exponents, if the bases are the same you subtract the exponents

$\frac{y^5}{y^8} = \frac{y \cdot y \cdot y \cdot y \cdot y}{y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y} = \frac{1}{y^3} = y^{5-8} = y^{-3}$

Applying the Rule #1: $x^a \div x^b = x^{a-b}$

$x^5 \div x^3 = \frac{x^5}{x^3} = \frac{x * x * x * x * x}{x * x * x} = x^2$

$x^7 \div x^2 = \frac{x^7}{x^2} = \frac{x * x * x * x * x * x * x}{x * x} = x^5$

$x^{12} \div x^3 = x^9 = x^{12-3}$

$\frac{x^9}{x^3} = x^6 = x^{9-3}$

$x^5 \div x^8 = x^{-3}$

$x^4 \div x^4 = x^0$

Zero Exponents

$x^3 \div x^3 = \frac{x^3}{x^3} = \frac{x * x * x}{x * x * x} = x^{3-3} = x^0 = 1$

Apply the division rule: $x^3 \div x^3 = x^{3-3} = x^0$

$y^5 \div y^5 = \frac{y^5}{y^5} = \frac{y * y * y * y * y}{y * y * y * y * y} = 1$

Apply the division rule: $y^5 \div y^5 = y^{5-5} = y^0$

Rule #2:

$x^0 =$ 1

Any base raised to a power of zero (with an exponent of zero) is equal to one

$(-x)^0 = 1$

$-[x^0] \neq 1$

$-[x]^0 = -[1] = -1$

Apply Rule #2: $x^0 = 1$

$x^{-5} \div x^{-5} =$ $x^{-5+5} = x^0 = 1$

$x^7 \div x^7 =$ 1

$7^0 =$ 1

$3x^0y =$ $3 \cdot 1 \cdot y = 3y$

$(3xy)^0 =$ 1

Remember: Any unstated exponent is assumed to have a value of 1

Note: In simplifying exponential expressions do not leave any "0" exponents - convert to "1".

Negative Exponents

$x^3 \div x^5 = \frac{x^3}{x^5} = \frac{x * x * x}{x * x * x * x * x} = \frac{1}{x^2}$

Apply the division rule: $x^3 \div x^5 = x^{3-5} = x^{-2}$

$y^4 \div y^9 = \frac{y^4}{y^9} = \frac{y * y * y * y}{y * y * y * y * y * y * y * y * y} = \frac{1}{y^5}$

Apply the division rule: $y^4 \div y^9 = y^{4-9} = y^{-5}$

Rule #3:

$$x^{-b} = \frac{1}{x^b}$$

Any base raised to a negative exponent is equivalent to its reciprocal

Applying Rule #3: $x^{-b} = \frac{1}{x^b}$

Remember: Any unstated exponent is assumed to have a value of 1

$x^{-3} =$ $\frac{1}{x^3}$

$4y^{-2}x =$ $\frac{4x}{y^2}$

$x^5 \div x^{-8} =$ $x^{5 \div \frac{1}{x^8}}$ x^{13}

$x^7 \div x^{12} =$ x^{-5} $\frac{1}{x^5}$

$x^{-4} \div x^{-8} =$ x^{-4+8} x^4

$\frac{1}{w^{-4}} =$ w^4 $\frac{w^0}{w^{-4}} = w^{0--4} = w^4$

Note: In simplifying exponential expressions do not leave any negative exponents - convert to positives

$\frac{1}{w^{-4}} = \frac{1}{\frac{1}{w^4}} = 1 \cdot \frac{w^4}{1} = w^4$

You try it:

$\frac{1}{2^0} =$ 1

$(-4)^{-3} =$ $\frac{1}{(-4)^3}$ $-\frac{1}{64}$

$4n^{?} =$ $\frac{4}{n^2}$ $?=-2$

$\frac{a^{?}}{3b^{?}} =$ $\frac{b^3}{3}$ $?1=0$ $?2=-3$

Note: In simplifying exponential expressions do not leave any zero or negative exponents.

$3ab^0 =$ 3a

$\frac{5^{-2}}{p} =$ $\frac{1}{25p}$

$\frac{6a^{-1}c^{-3}}{d^0} =$ $\frac{6}{ac^3}$

$9^0y^7t^{-11} =$ $\frac{y^7}{t^{11}}$

$\frac{1}{1000} = 10^{?}$ -3

Note: In simplifying exponential expressions do not leave any zero or negative exponents.

$100{,}000 = 10^{?}$ 10^5

$10^{-6} =$ $\frac{1}{1{,}000{,}000}$

$10^4 =$ 10,000

$2^3(5^0 - 6m^2)$ $8(1-6m^2)$ $8-48m^2$

$\frac{6}{m^2} + \frac{5m^{-2}}{3^{-3}}$ $= \frac{36}{3^2} + \frac{3^3 \cdot 5}{3^2} = \frac{141}{3^2}$

$\frac{2r^{-5}y^3}{n^2} \div \frac{r^2y^5}{2n} = \frac{2r^{-7}y^{-2}}{n} = \frac{4}{r^7y^2n^1}$

$\frac{2y^3}{n^2r^5} \div \frac{r^2y^5}{2n} = \frac{2y^3}{n^2r^5} \cdot \frac{2n}{r^2y^5} = \frac{4y^3n}{n^2r^7y^5y^2}$

$\frac{4y^{-2}n^{-1}}{r^7} = \frac{4}{nr^7y^2}$

$$\frac{2^{-4} r^{3} s^{-2}}{1}$$

$$\frac{r^3}{2^4 s^2} = \frac{r^3}{16 s^2}$$