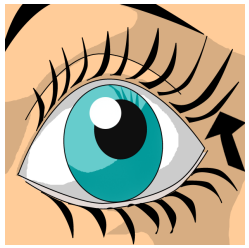
6-5:
Linear Inequalities
&
6-6:
Systems of Linear Inequalities

Eye Opener



You are buying paperback and hardcover books at a book sale. You can spend at most $20. What are the possible combinations of paperback and hardcover books that you can buy? Explain.

You want to buy at least 6 new ring tones from a Web site, but you cannot spend more than $15. How many premium ring tones and how many top-10 ring tones can you buy? Explain how you found your answer.

Essential Understanding A linear inequality in two variables has an infinite number of solutions. These solutions can be represented in the coordinate plane as the set of all points on one side of a boundary line.

You can graph the solutions of a system of linear inequalities in the coordinate plane. The graph of the system is the region where the graphs of the individual inequalities overlap.

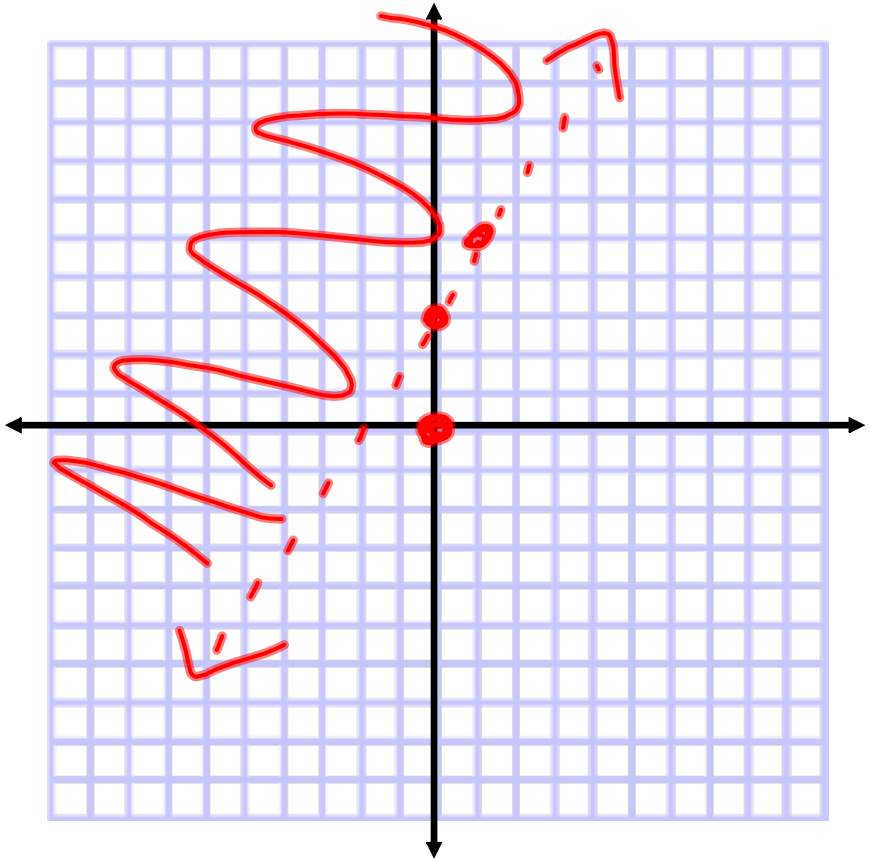
A **system of linear inequalities** is made up of two or more linear inequalities. A **solution of a system of linear inequalities** is an ordered pair that makes *all* the inequalities in the system true. The graph of a system of linear inequalities is the set of points that represent all of the solutions of the system.

Linear inequalities are similar to one-variable inequalities:

- One varaible inequalities are bounded by dots/endpoints on a number line
  - Two variable inequalities are bounded by lines on a coordinate grid
- One variable inequalities use a shaded dot to indicate the point is included
  - Two variable inequalities use a solid line to indicate the line is included
- One variable inequalities use an unshaded endpoint to indicate the point is excluded
  - Two variable inequalities use a dashed/broken line to indicate the line is excluded
- One variable inequalities use shading between/around the dots/endpoints to indicate the values included in the solution set
  - Two variable inequalities use shading between/around the lines to indicate the values included in the solution set
- One-variable inequalities shade to the left for less than/less than or equal to ($<$ and $\leq$)
  - Two-variable inequalities in slope-intercept form shade below the line for less than/less than or equal to ($<$ and $\leq$)
- One-variable inequalities shade to the right for greater than/greater than or equal to ($>$ and $\geq$)
  - Two variable inequalities in slope-intercept form shade above the line for greater than/greater than or equal to ($>$ and $\geq$)

Linear inequalities are different from one-variable inequalities:

- One variable inequalities can only be graphed by hand
  - Two variable inequalities can be graphed by hand OR by calculator



Graph $y > 2x + 3$

$0 > 2(0) + 3$

$0 \not> 3$

$<$ $>$ broken

$\leq$ $\geq$ solid

$y < mx + b$ shade ↓

$y > mx + b$ shade ↑

Graph: $3x - 2y < -4$

$(0, 2)$

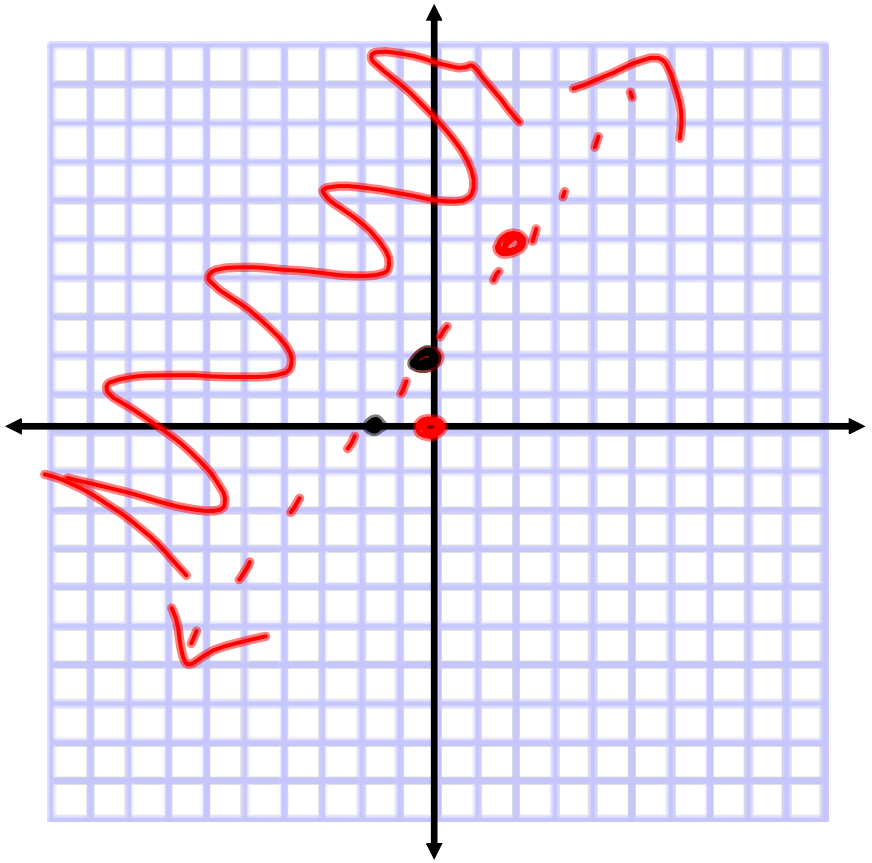
$(-\frac{4}{3}, 0)$

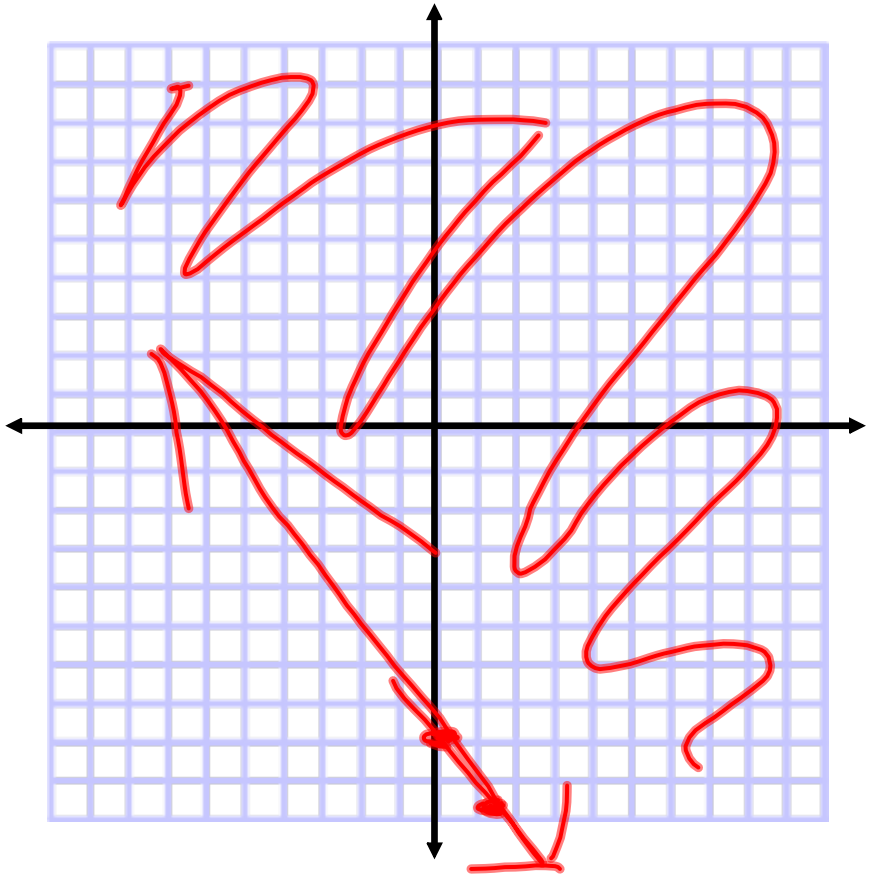
$\frac{-2y}{-2} < \frac{-3x-4}{-2}$

$y > \frac{3}{2}x + 2$

$3(0) - 2(0) < -4$

$0 \not< -4$





Graph: $y \geq -2x - 8$

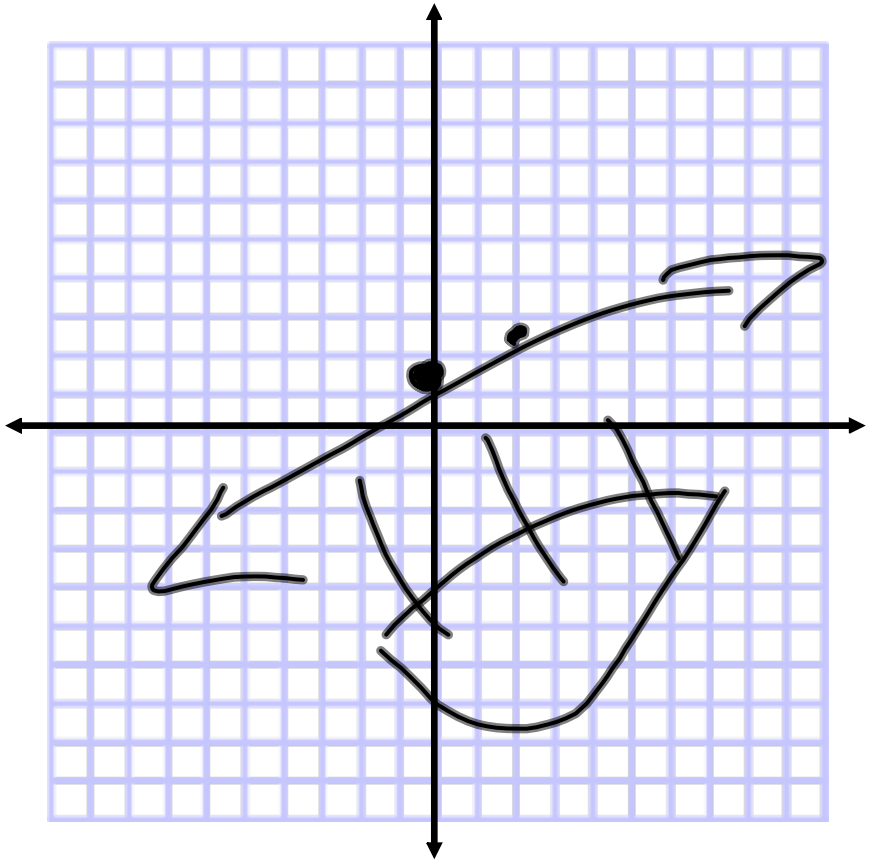
Graph: $-x + 2y \leq 3$

$+x \quad +x$

$\frac{2y}{2} \leq \frac{x+3}{2}$

$y \leq \frac{1}{2}x + \frac{3}{2}$

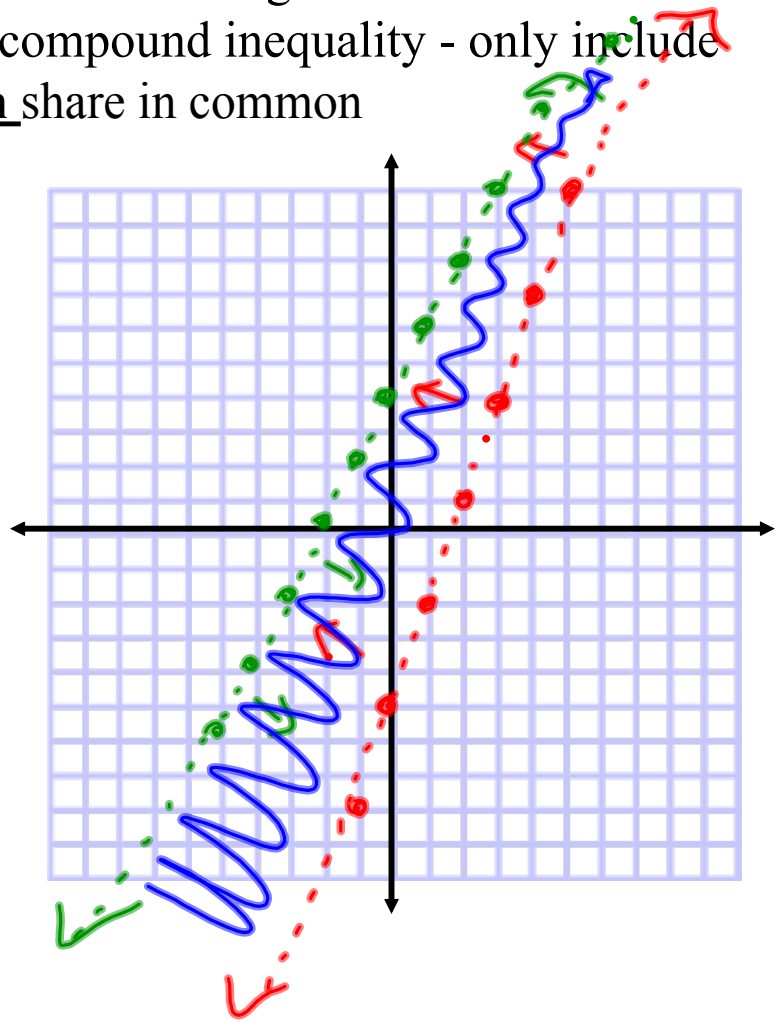
$y \leq \frac{1}{2}x + 1.5$



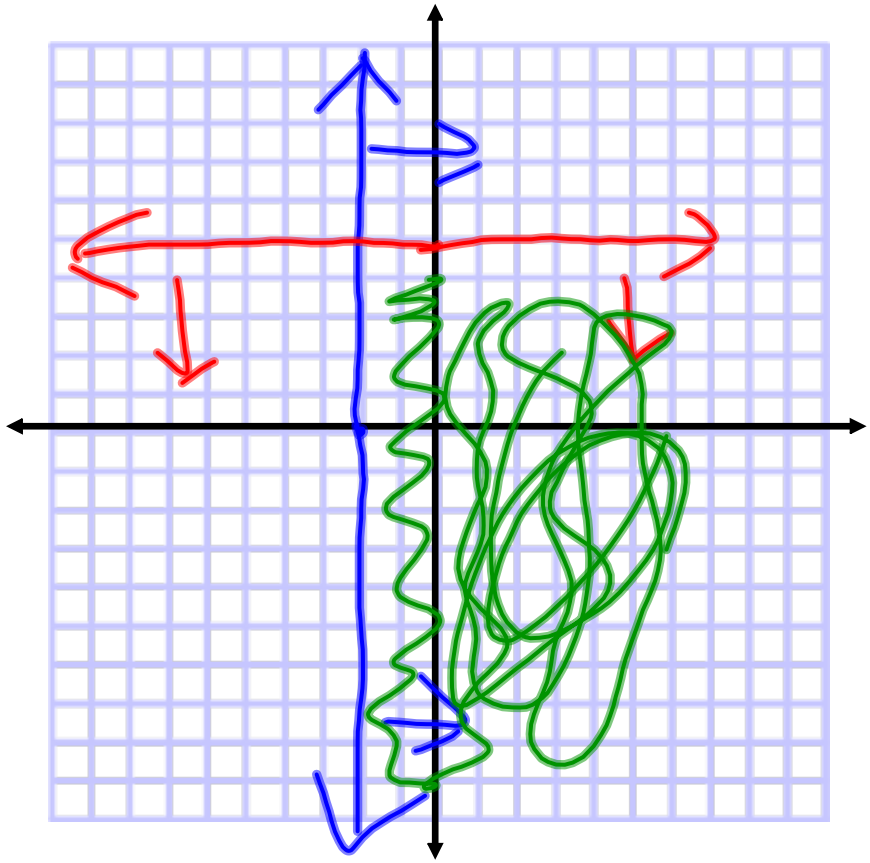
Solving Systems of Linear Inequalities

- Can only be solved by graphing
- Graph both inequalities on the same coordinate grid
- Treat like an "AND" one-variable compound inequality - only include the points / shaded regions they **both** share in common

Solve: $y > 3x - 5$ and $y < 2x + 4$



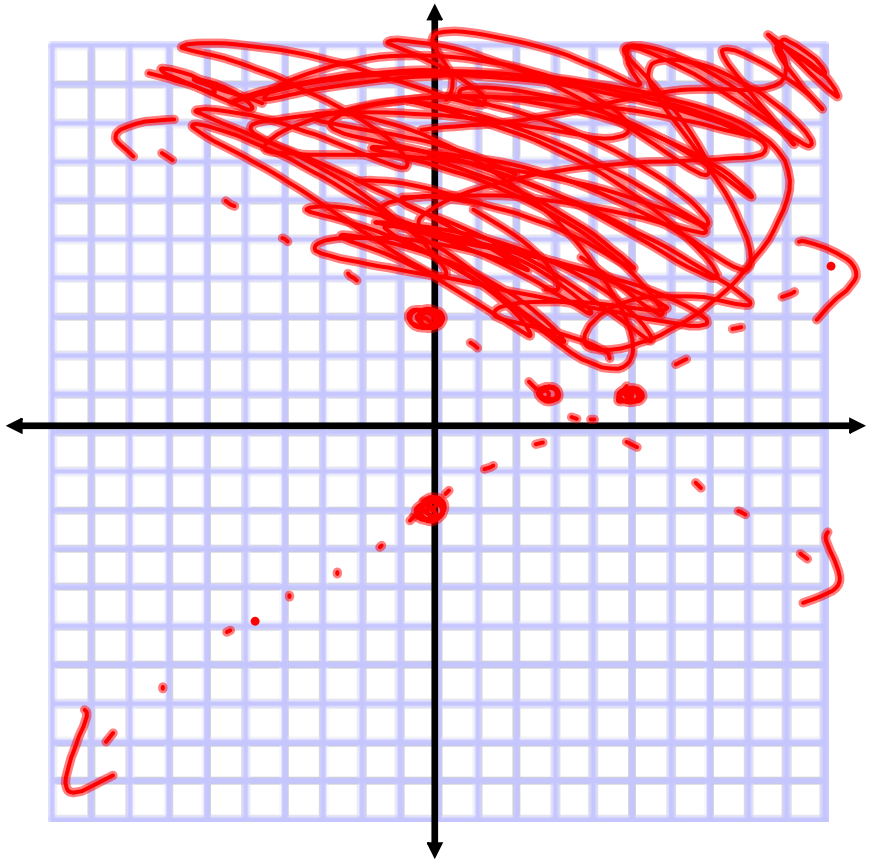
Solve: $x \geq -2$ and $y \leq 5$



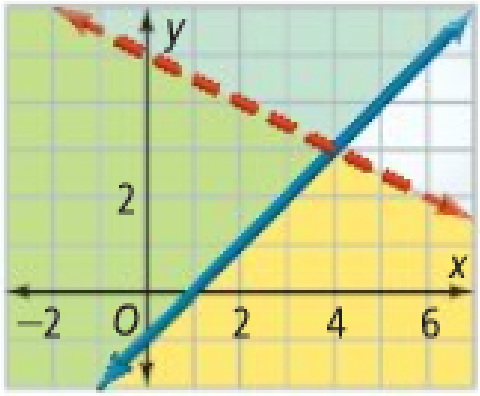
Solve: $2x + 3y > 9$
$3x - 5y < 10$

$y > \frac{-2}{3}x + 3$

$y > \frac{3}{5}x - 2$



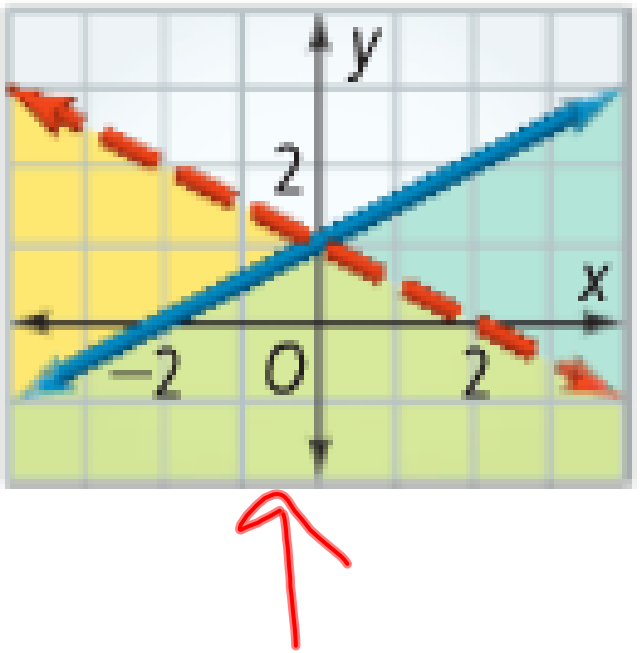
What system of inequalities is represented by the graph below?



A) $y < -\frac{1}{2}x + 5$

B) $y \geq x - 1$

What system of inequalities is represented to the right?



A) $y < -\frac{1}{2}x + 1$

B) $y \leq \frac{1}{2}x + 1$

Reasonable Restrictions on Word Problems Involving Systems of Equations

Time Management You are planning what to do after school. You can spend at most 6 h daily playing basketball and doing homework. You want to spend less than 2 h playing basketball. You must spend at least $1\frac{1}{2}$ h on homework. What is a graph showing how you can spend your time?

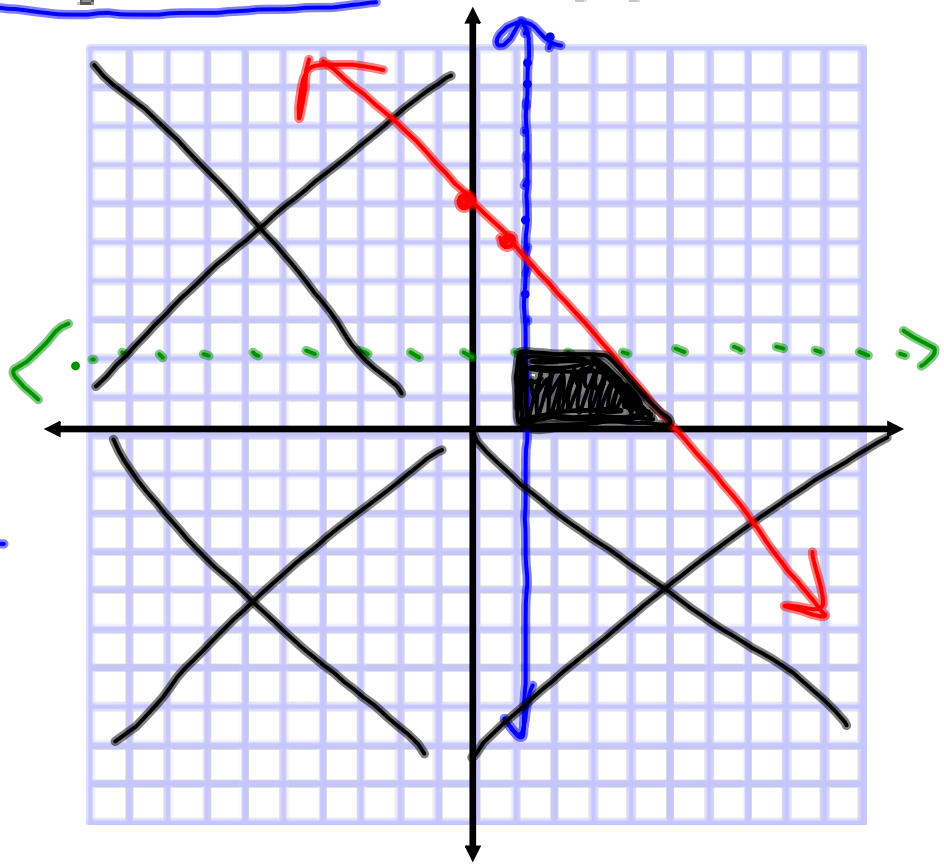
x = homework time

y = basketball time

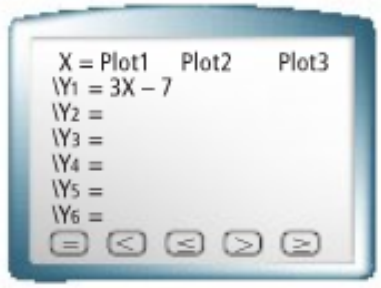
$x \geq 0 \quad y \geq 0$

$y + x \leq 6$

$y < 2 \quad x \geq 1.5$



A graphing calculator can show the solutions of an inequality or a system of inequalities. To enter an inequality, press **apps** and scroll down to select **INEQUAL**. Move the cursor over the $=$ symbol for one of the equations. Notice the inequality symbols at the bottom of the screen, above the keys labeled **F2–F5**. Change the $=$ symbol to an inequality symbol by pressing **alpha** followed by one of **F2–F5**.



Activity 1

Graph the inequality $y < 3x - 7$.

1. Move the cursor over the $=$ symbol for Y_1. Press **alpha** and **F2** to select the $<$ symbol.
2. Enter the given inequality as Y_1.
3. Press **graph** to graph the inequality.

Activity 2

Graph the system. $y < -2x - 3$
$y \geq x + 4$

4. Move the cursor over the $=$ symbol for Y_1. Press alpha and F2 to select the $<$ symbol. Enter the first inequality as Y_1.
5. Then move the cursor over the $=$ symbol for Y_2, and press alpha and F5 to select the $\geq$ symbol. Enter the second inequality as Y_2.
6. Press graph to graph the system of inequalities.