

# Using Matrices to Solve Systems of Equations

## Essential Understanding

There is more than one way to solve a system of equations. Some systems are written in a way that makes the graphing calculator and matrices a good method to use.



A *matrix* is a rectangular arrangement of numbers in rows and columns. The plural of *matrix* is *matrices*.

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You can use a special type of matrix, called an *augmented matrix*, to solve a system of linear equations. An augmented matrix is formed using the coefficients and constants in the equations in a system. The equations must be written in standard form.

### System of Equations

$$7x + 6y = 10$$

$$4x + 5y = -5$$

### Augmented Matrix

$$\left[ \begin{array}{cc|c} 7 & 6 & 10 \\ 4 & 5 & -5 \end{array} \right]$$

Goal: Use matrix multiplication to solve systems of equations on the graphing calculator.

To Multiply two matrices:

- Multiply the elements in a row of the first matrix by the corresponding elements in a column of the second matrix, then add the products.
- The number of columns in the first matrix must equal the number of rows in the multiplying matrix.

$$\begin{array}{c}
 \text{3 columns} \\
 \begin{array}{c} \swarrow \downarrow \downarrow \\ \left[ \begin{array}{ccc} 2 & 5 & -3 \\ 3 & 1 & 6 \end{array} \right] \left[ \begin{array}{c} 7 \\ 2 \\ 1 \end{array} \right] \\
 \leftarrow \leftarrow \leftarrow \text{3 rows}
 \end{array}
 \end{array}$$

$$\begin{aligned}
 &= 2(7) + 5(2) + (-3)(1) = 14 + 10 - 3 = \begin{bmatrix} 21 \\ 29 \end{bmatrix} \\
 &= 3(7) + 1(2) + 6(1) = 21 + 2 + 6 = \begin{bmatrix} 21 \\ 29 \end{bmatrix}
 \end{aligned}$$

Use Matrix Multiplication:

$$[4 \ 2 \ 9] \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} \text{ means } 4(3) + 2(1) + 9(7) = 12 + 2 + 63 = 77$$

Use the graphing calculator: Access the MATRIX key

Let Matrix A =  $[4 \ 2 \ 9]$  = 1 x 3 matrix

Let Matrix B =  $\begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$  = 3 x 1 matrix

Set dimensions of Matrix A: 1 x 3 and enter content  $[4 \ 2 \ 9]$

Exit

Set dimensions of Matrix B: 3 x 1 and enter content  $[3 \ 1 \ 7]$

Exit

From home screen select MATRIX and  $[A] [B]$

Results in the multiplication of  $[A] * [B] = [77]$

You try:

$$\begin{bmatrix} 12 & 10 \\ 8 & -11 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 9 & -1 \end{bmatrix}$$

$$\begin{aligned} 12(0) + 10(9) &= 90 \\ 12(4) + 10(-1) &= 38 \\ 8(0) + -11(9) &= -99 \\ 8(4) + -11(-1) &= 43 \end{aligned}$$

$$\begin{bmatrix} 2 & 5 & -3 \\ 3 & 1 & 6 \end{bmatrix} \begin{bmatrix} 7 & 3 \\ 2 & 9 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 45 \\ 29 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 44 & -12 \\ 27 & 35 \\ 25 & -16 \end{bmatrix} \begin{bmatrix} 21 & -41 \\ 25 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 624 & -2008 \\ 1442 & -512 \\ 125 & -129 \end{bmatrix}$$

$$\left. \begin{aligned} 44(21) + -12(25) \\ 44(-41) + -12(17) \end{aligned} \right\}$$

$$\begin{bmatrix} 212 & 318 & 175 \\ 185 & 292 & 221 \end{bmatrix} \begin{bmatrix} .96 \\ 1.23 \\ 1.51 \end{bmatrix}$$

Concept: Systems of Equations in "Semi-Standard" ( $Ax + By = C$  where  $A$ ,  $B$ , &  $C$  are any numbers) form lend themselves to matrix multiplication.

$$2x + 6y = 80$$

$$4x + 5y = -1$$

$$\begin{bmatrix} 2 & 6 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 80 \\ -1 \end{bmatrix}$$

$$\begin{array}{ccccc} \text{Matrix} & & \text{Matrix} & & \text{Matrix} \\ A & * & B & = & C \end{array}$$

- To solve for Matrix B divide both sides by Matrix A
- Matrix B = Matrix C / Matrix A
- Division = Multiplication by the Reciprocal
- Matrix B = Matrix C \* 1/Matrix A = Matrix C \* Matrix ~~A~~<sup>-1</sup>

Using the Calculator:

$$2x + 6y = 80$$

$$4x + 5y = -1$$

- Click the MATRIX key
- Edit the dimensions: Row x Column
- $A = 2 \times 2$        $C = 2 \times 1$
- Edit the content:  $A = \begin{matrix} 2 & 6 \\ 4 & 5 \end{matrix}$        $C = \begin{matrix} 80 \\ -1 \end{matrix}$
- Use the MATRIX key. Select [C] then [A] then  $x^{-1}$  then press ENTER
- The calculator will calculate the values of  $x$  &  $y$ , in this case,  $\{-29$   
 $[23]\}$

You try:

$$x + 8y = 16$$

$$9x + 12y = 66$$

$$29x + 7y = 1012$$

$$8x - 25y = 737$$

$$76x + 18y = 86$$

$$189x + 47y = 132$$

$$x - 3y = 8$$

$$y = x - ?$$

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$$\begin{array}{r} x - 3y = 8 \\ -x + y = -? \end{array}$$

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$$\begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 \\ -? \end{bmatrix}$$

$$1.285x - 3.75y = 29.23$$

$$3.759x + 4.28y = 18.53$$

$$= \begin{pmatrix} -6.469752049 \\ 10.01163504 \end{pmatrix}$$
$$= \begin{pmatrix} 9.930159394 \\ -4.391932048 \end{pmatrix}$$