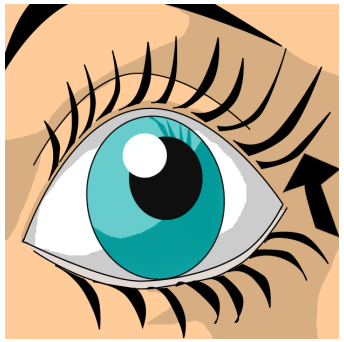
6-3:
Solving Systems of Equations Using Elimination (Simultaneous Equations)

Eye Opener



x = wt. apple y = wt orange

A cafeteria sells fresh fruit by weight. All apples weigh the same, and all oranges weigh the same. What is the weight of an apple? What is the weight of an orange? How do you know?

$$-(2x+3y=37)$$
$$2x+5y=51$$
$$2y=14$$

$2y = 14$
$y = 7$
$x = 8$

Essential Understanding There is more than one way to solve a system of equations. Some systems are written in a way that makes eliminating a variable a good method to use.

Elimination Method: You can use the Add/Sub/Mult/Div Properties of Equality to solve a system of equations, by adding *(subtracting)* the equations to eliminate a variable.

Pros:

- Does not involve graphing & guessing & manipulating
- Direct Solution
- Does not involve converting equations to other forms

Cons:

- Sometimes you must multiply one or both equations by a value to create variables that are opposites
- Careless error: Forgetting to distribute properly
- Careless error: Sign errors

Elimination Process:

1. Check the coefficients of your variables; are any of them opposites?
If so, go to step 3.
2. Multiply **both sides** of one or both equations so that one pair of variables become opposites.
3. Add the two equations together eliminating one variable.
4. Solve the remaining one-variable equation.
5. Substitute the value found in step 4 into one of the **original** equations to find the other variable.
6. Check the newly found ordered pair (x,y) in the other **original** equation.

Solve using Elimination:

$5x - 6y = -32$
$3x + 6y = 48$

$8x = 16$
$x = 2$

✓ Are either variables opposites? yes $-6y, 6y$

✓ If yes, add equations.

~~If~~ not multiply equations to make opposites.

✓ Solve the one variable equation

✓ Substitute to find other variable

✓ Check answer in the **original** equations

$5x - 6y = -32$
$5(2) - 6y = -32$
$10 - 6y = -32$
$-10 \quad -10$
$-6y = -42$
$\frac{-6y}{-6} = \frac{-42}{-6}$
$y = 7$

$(2, 7)$

Solve using Elimination:

$2x + 5y = -22$
$10x + 3y = 22$

Are either variables opposites? No

If yes, add equations.

If not multiply equations to make opposites, then add equations.

$-5(2x + 5y = -22)$
$-10x - 25y = 110$
$10x + 3y = 22$

$-22y = 132$
$-22 \quad -22$

$y = -6$

Solve the one variable equation

Substitute to find other variable

$2x + 5y = -22$
$2x + 5(-6) = -22$
$2x - 30 = -22$
$+30 \quad +30$
$2x = 8$
$x = 4$

Check answer in the **original** equations

$2x + 5y = -22$
$8 + -30 \stackrel{?}{=} -22$ ✓

$10x + 3y = 22$
$10(4) + 3(-6) = 22$
$40 + -18 = 22$ ✓

$(4, -6)$

Solve using Elimination:

$4x + 2y = 14$

$7x - 3y = -8$

Are either variables opposites?

If yes, add equations.

If not multiply equations to make opposites, then add equations.

Solve the one variable equation

Substitute to find other variable

Check answer in the **original** equations

$(1, 5)$

You sell 292 tickets to a basketball game. Adult tickets cost $3 and student tickets cost $1. You collect $470 dollars in ticket sales. How many of each ticket type did you sell?

Solve using two variables:

(89, 203)

	# · $	= T+1 $
A	x	3	3x
S	x	1	y
T+1	292	~	470

$x + y = 292$

$3x + y = 470$

$2x = 118$

$x = 89$

$292 - 89 = 203 = y$

A weekend at a hotel costs $195 for 2 nights and 4 meals. The same hotel books a week stay at $650 for 7 nights and 10 meals. What is the cost of a meal and the cost per night?

Solve using two variables:

x = \$ night
y = \$ meal

	#nts	#meals	cost
opt 1	$2x$	$4y$	195
opt 2	$7x$	$10y$	650

$$7(2x+4y=195) \Rightarrow 14x+28y=1365$$
$$-2(7x+10y=650) \Rightarrow -14x-20y=-1300$$
$$8y=65$$
$$y=8$$

You try:

$$6x - 3y = 3$$
$$-6x + 5y = 3$$

$$-2x + 15y = -32$$
$$7x - 5y = 17$$

$$15x + 3y = 9$$
$$10x + 7y = -4$$

Your class sells a total of 64 tickets to a play. Student tickets cost $1 and adult cost $2.50. If you collect a total of $109 in sales, how many of each ticket did you sell?

Does this problem require two variables?

Solve using two variables:

The photography company offers two packages for school pictures. Plan A includes 30 wallets and 1 - 8 x 10 for $17.65. Plan B includes 20 wallets and 3 - 8 x 10's for $25.65. Find the cost of wallets and of 8 x 10's.

Does this problem require two variables?

Solve using two variables.

$y \geq -\frac{4}{5}(x-1)$

$y \leq \frac{5}{6}(x-1)$

$(1,0)$

$y \geq -\frac{4}{5}x + \frac{4}{5}$

$y \leq 5/6x - 5/6$

$y+4 \geq -\frac{4}{5}(x-6)$

$y+5 \leq 5/6(x+5)$

x = length

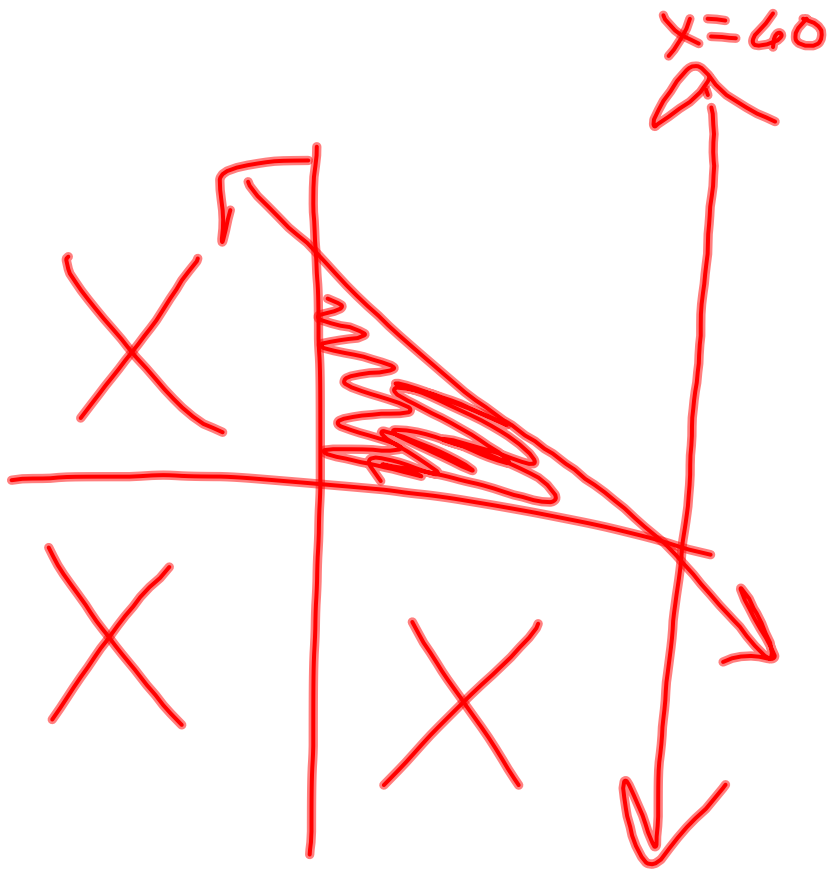
y = width

$x \leq 60$

$2x+2y \leq 136$

✓ $x \geq 0$

✓ $y \geq 0$



x = # babysit ($5)

y = # bagging ($6)

$x \geq 0$

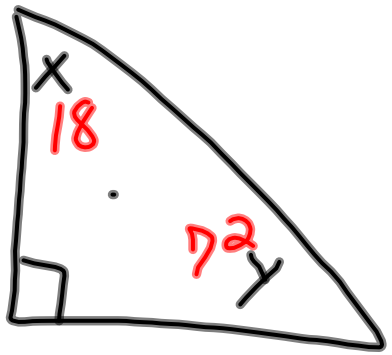
$y \geq 0$

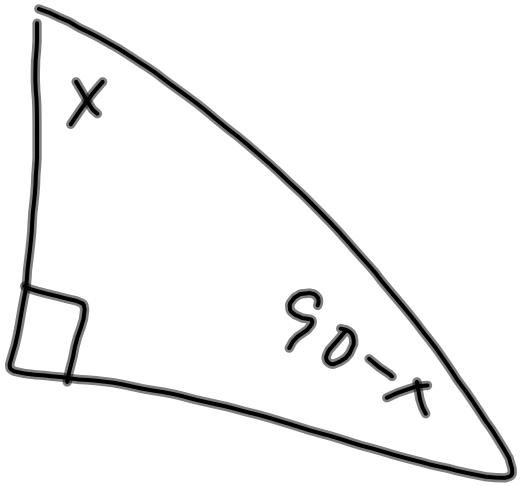
$x + y \leq 20$

$5x + 6y \geq 90$

buses and how many vans will be needed.

25. Geometry The measure of one acute angle in a right triangle is four times the measure of the other acute angle. Write and solve a system of equations to find the measures of the acute angles.





$x + y = 90$

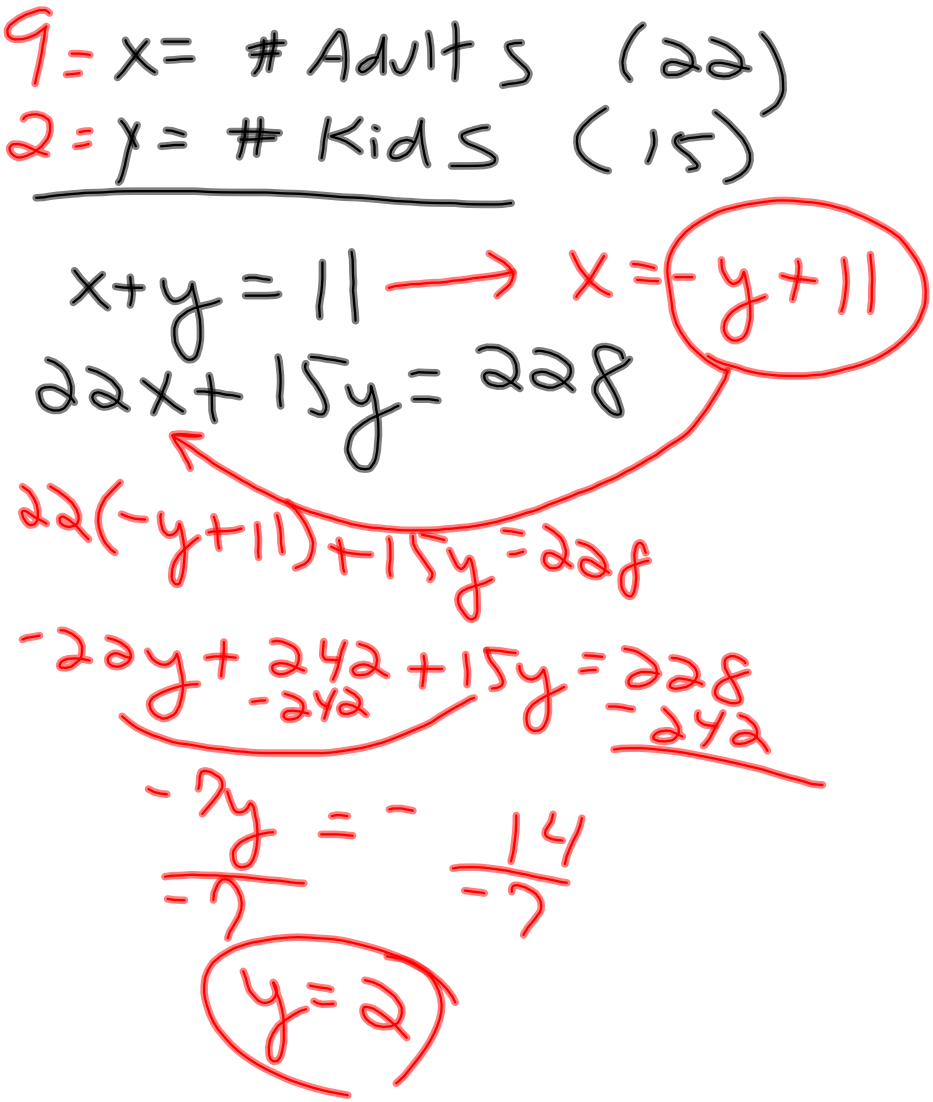
$y = 4x$

$x + 4x = 90$

$5x = 90$

$x = 18$

23. Theater Tickets Adult tickets to a play cost $22. Tickets for children cost $15. Tickets for a group of 11 people cost a total of $228. Write and solve a system of equations to find how many children and how many adults were in the group.



24. Transportation A school is planning a field trip for 142 people. The trip will use six drivers and two types of vehicles: buses and vans. A bus can seat 51 passengers. A van can seat 10 passengers. Write and solve a system of equations to find how many buses and how many vans will be needed.

x = # buses (51)
y = # vans (10)
} 6 drivers

Total: 142 people

2 buses
4 vans

$x + y = 6 \rightarrow y = -x + 6$
$51x + 10y = 142$

$51x + 10(-x + 6) = 142$
$51x - 10x + 60 = 142$
$-60 \quad -60$
$\frac{41x}{41} = \frac{82}{41}$
$x = 2$

36. Art An artist is going to sell two sizes of prints at an art fair. The artist will charge \$20 for a small print and \$45 for a large print. The artist would like to sell twice as many small prints as large prints. The booth the artist is renting for the day costs \$510. How many of each size print must the artist sell in order to break even at the fair?

(21

The systen
solutions.

x = # small prints (20) - 12

y = # lrg prints (45) - 6

$x = 2y$

$510 = 20x + 45y$

$510 = 20(2y) + 45y$

$510 = 40y + 45y$

$\frac{510}{85} = \frac{85y}{85}$

$6 = y$

30. **Nutrition** Half a pepperoni pizza plus three fourths of a ham-and-pineapple pizza contains 765 Calories. One fourth of a pepperoni pizza plus a whole ham-and-pineapple pizza contains 745 Calories. How many Calories are in a whole pepperoni pizza? How many Calories are in a whole ham-and-pineapple pizza?

43. $2x - 3y + z = 0$
$2x + y + z = 12$
$y - z = 4$

$y = 2x + 2$ $y = -3x + 4$

$3x + y = 4$

$2x + 2 = -3x + 4$

$+3x \quad -2 \qquad +3x - 2$

$5x = 2$

$x = 2/5$ or $.4$

$3x + 2x + 2 = 4$

$5x + 2 = 4$

$5x = 2$

$x = 2/5$

$y = 2.8$

$(.4, 2.8)$

x = # vans

y = # cars

$$x + y = 5$$
$$7x + 5y = 31$$