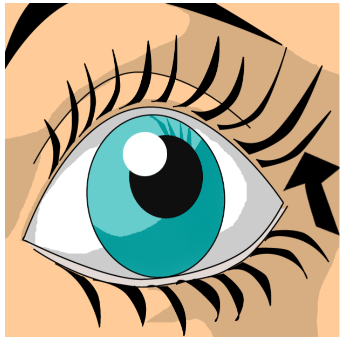
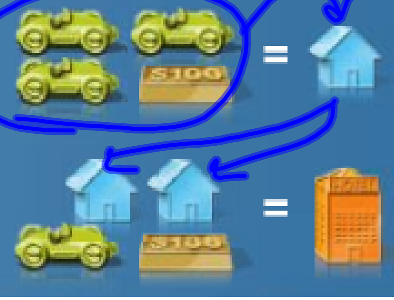
6-2: Solving Systems of Linear Equations Using the Substitution Method

Eye Opener



A board game allows players to trade game pieces of equal value. The diagram shows two fair trades. The hotel is worth \$2400. How much is a car worth? Explain your reasoning.



$C = 300$

$7C = 2100$

$7C + 300 = 2400$

$3C + 100 = H$

$\cancel{2H} + 1C + 100 = 2400$

$(3C+100)+(3C+100)+1C+100 = 2400$

Solve each equation for y.

$4x + 2y = 38$ $\qquad$ $\frac{1}{2}x + \frac{1}{3}y = 5$ $\qquad$ $\frac{3}{2}y = \frac{4}{5}x$ $\qquad$ $1.5x - 4.5y = 21$

$y = -2x + 19$ $\qquad$ $y = -\frac{3}{2}x + 15$ $\qquad$ $y = \frac{8}{15}x$ $\qquad$ $y = \frac{1}{3}x - 4.\overline{6}$

$x = 10 - \frac{2}{3}y$ $\qquad$ $x = \frac{15}{8}y$ $\qquad$ $x = 3y + 14$

$x = 9\frac{1}{2} - \frac{1}{2}y$

Essential Understanding Systems of equations can be solved in more than one way. When a system has at least one equation that can be solved quickly for a variable, the system can be solved efficiently using substitution.

Substitution Method: By replacing one variable with an equivalent expression containing the other variable, you can create a one-variable equation that you already know how to solve.

<u>Pros:</u>
- Gives you a "non-graphing" method to use for solving a system of equations
- Gives you a "direct" mathematical approach to use to solve a system of equations

<u>Cons:</u>
- Requires that at least one equation be solved for "x" or "y" (slope-intercept form)
- Requires an "interpretation" of resulting 1-variable equation solutions like:

3 = 3	OR	5 = -8
infinite solutions		no solutions
same lines		parallel lines

- Once you solve for one variable you have to go back and solve for the other variable

Solve using Substitution: $y = -4x + 8$ and $y = x + 7$

Since they are both solved for "y", we substitute one "y" for the other becasue we want to know when their y values will be the "same" (their point of intersection)

y = "-4x + 8" substituted into "y" = x + 7 results in

$-x \quad -8 \quad -x \quad -8$

$-4x + 8 = x + 7$ Finish Solving for "x"

$-5x = -1$

$x = 1/5$

$(1/5, 7\,1/5)$

Once you have solved for "x", you must go back and solve for "y" because we are looking for the **point** of intersection which requires an ordered pair (x,y)

Do that by substituting the value for "x" into one of the original equations to solve for "y" and then check the resulting ordered pair in the other equation

Solve for y in: $y = -4x + 8$ and Check (x,y) in: $y = x + 7$

$y = -4\left(\frac{1}{5}\right) + 8$

$y = \frac{-4}{5} + 8$

$y = 7\,1/5$

$y = \frac{1}{5} + 7$

$y = 7\,1/5$

Solve using substituiton: $3y + 2x = 4$ and $-6x + y = -7$

1. Solve for x or y in one equation, if needed (choose the easier of the two to solve for)

$-6x+y=-7$

$y=6x+{}^{-}7$

2. Substitute this variable's "literal" solution into the other equation and solve the resulting **one** variable equation.

$3y+2x=4$

$3(6x-7)+2x=4$

$18x-21+2x=4$

$20x=25$

$x=\frac{25}{20}=\frac{5}{4}$ or 1.25

3. Substitute the solved variable into either **original** equation to find the remaining variable.

$3y+2x=4$

$3y+2(1.25)=4$

$3y+2.5=4$

$3y=1.5$

$y=.5$

$-6x+y=-7$

$-6(1.25)+y=-7$

$-7.5+y=-7$

$y=.5$

$(1.25, .5)$

4. Check your ordered pair in the unused **original** equation in step 3.

Solve using the substitution method:

You are planning a trip for 193 people. There are eight drivers available to drive 51-passenger buses and 8-passenger mini-vans. How many of each are needed?

Define two variables:

Write two equations:

Solve:

Match the system of equations with its solution.

$y = x + 1$ and $y = 2x - 1$ (3, 2)

$x+1 = 2x-1$
$-2x -1 \quad -2x -1$
$-x = -2$

$x = 2$
$y = 3$

(3, 3)

$2y = x + 3$ and $x = y$ (-2, 3)

(2, 3)

$2y = y + 3$
$y = 3$

$x = y$
$x = 3$

You try:

Solve using substitution:

$y = 4x - 8$ and $y = 2x + 10$

$y = 4(9) - 8$

$36 - 8$

$y = 28$

$(9, 28)$

$x = 9$

$$7x - 8y = 112 \quad \text{and} \quad y = -2x + 9$$

=

=

$7x - 8(-2x + 9) = 112$

$7x + 16x - 72 = 112$

$+72 \qquad +72$

$\frac{23x}{23} = \frac{184}{23}$

$x = 8$

$y = -2x + 9$

$y = -2(8) + 9$

$y = -16 + 9$

$y = -7$

$(8, -7)$

$4x + y = -2$ and $-2x - 3y = 1$

$-4x \qquad -4x$

$y = -4x + {}^{-}2$

$=$

$-2x + {}^{-}3(-4x - 2) = 1$

$-2x + 12x + 6 = 1$

$10x + 6 = 1$

$10x = {}^{-}5$

$x = -\frac{1}{2}$

$4x + y = -2$

$4(-\frac{1}{2}) + y = -2$

$-2 + y = -2$

$y = 2 + {}^{-}2$

$y = 0$

$\left(-\frac{1}{2}, 0\right)$

Which of these was the easiest to solve using substitution?

Which was the most difficult to solve using the substitution method?

$y = 4x - 8$ and $y = 2x + 10$ easiest

$4x - 8 = 2x + 10$

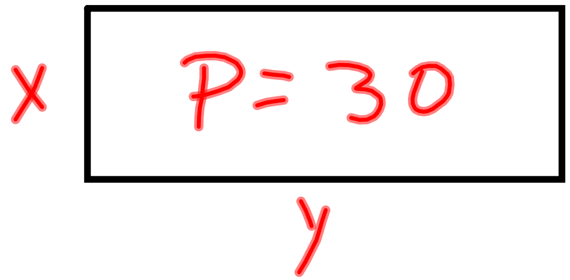
$7x - 8(y) = 112$ and $y = (-2x + 9)$

$4x + y = -2$ and $-2x - 3(y) = 1$ most difficult

$y = (-4x - 2)$

A rectangle is 4 times longer than it is wide. The perimeter of the rectangle is 30 inches. Find the dimensions of the rectangle.

x = width   y = length

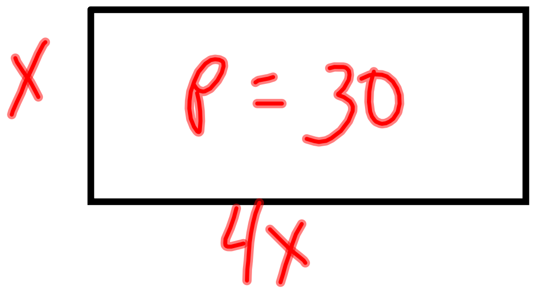


Solve using two variables:

$2x + 2y = 30$

$y = 4x$

Could it have been solved using one variable?



Yes

$2x + 2(4x) = 30$