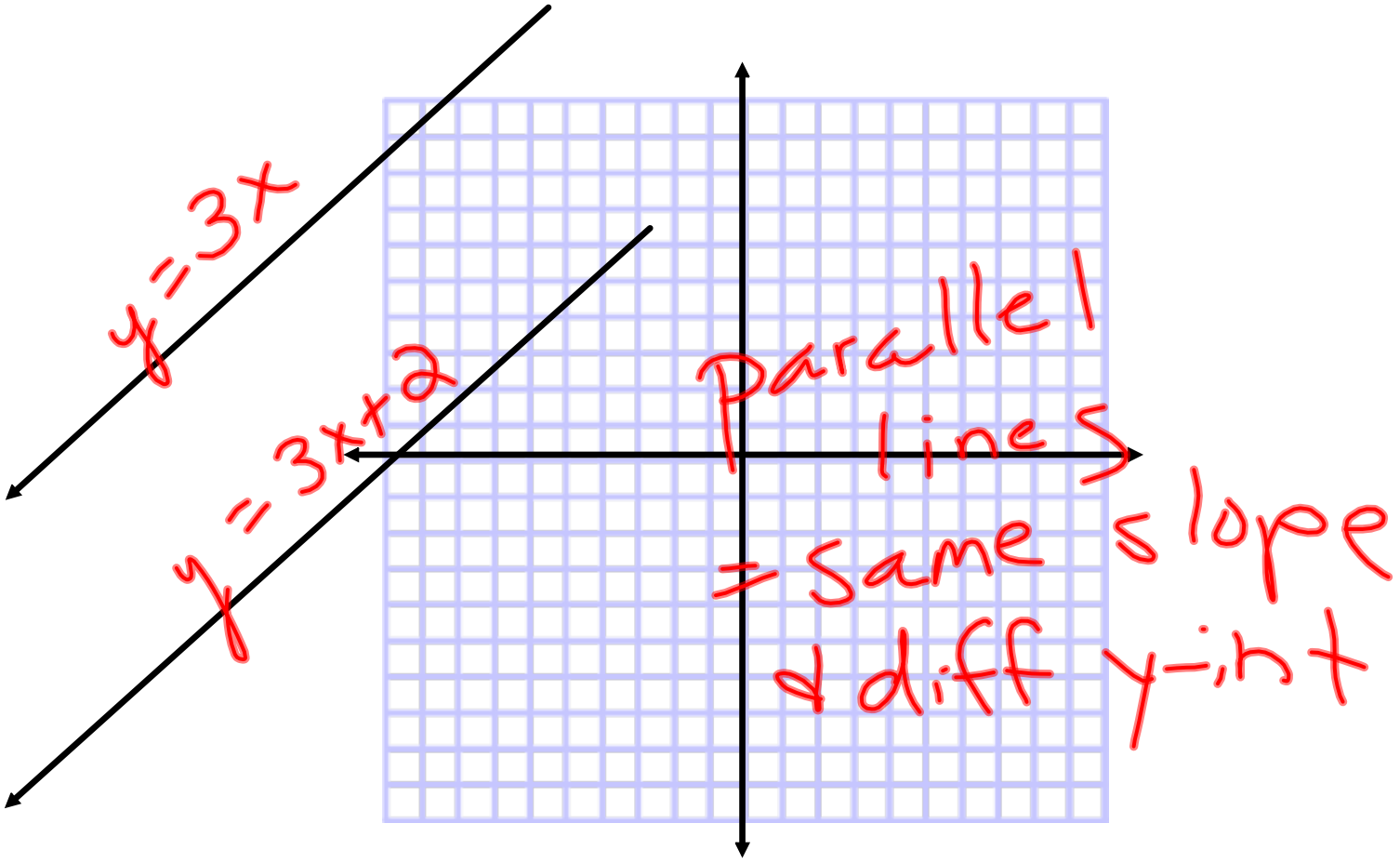
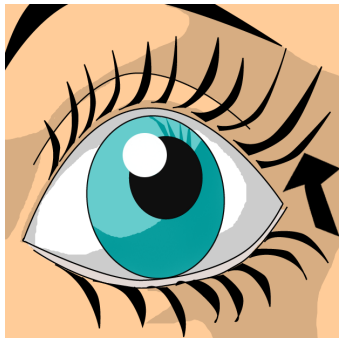
5-6: Parallel & Perpendicular Lines


y=3x
y=3x+2
Parallel lines
= same slope
& diff y-int

Intersecting Lines = diff slopes

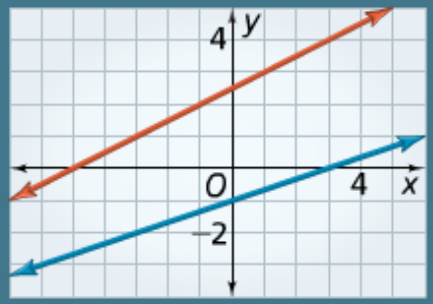
Eye Opener



Find the slope of the line that passes through each pair of points.

$(0, -4), (2, 0)$ $(5, 5), (3, -1)$ $(-4, 2), (5, 2)$

Copy the graph shown. Can you draw a line that will not intersect either of the lines in the graph? If so, draw the line. If not, why not? Can you draw a line that will intersect one of the lines in such a way that the intersection forms four congruent angles? If so, draw the line. If not, why not?



Essential Understandings

You can determine the relationship between two lines by comparing their slopes and y-intercepts.

Any two lines in a coordinate plane either intersect or are parallel.

You can use slope to determine if two lines are parallel or perpendicular.

Lines With the Same Slope

Parallel lines are lines in the same plane that never intersect.

Any two lines with the **same slope** and **different y-intercepts** are **parallel**

- all vertical lines are parallel
- all horizontal lines are parallel

Examples: $y = 3x + 5$ OR $y = -1/2x + 3$ OR $y = -7$
$y = 3x - 7$ $y = -1/2 - 8$ $y = 1/2$

Lines with the **same slope and same y-intercept** are the **same line** with a different name

$y = 3x + 5$
$y = 3x + 5$

$y - 2 = 3(x + 4)$
$y + 5 = 3(x - 7)$?

Lines with Different Slopes

Lines with **different slopes intersect**

Perpendicular lines are lines in the same plane that intersect to form right or 90 degree angles

Any two lines with slopes which are negative reciprocals of each other (**product of the two slopes is -1**) are perpendicular

- all horizontal and vertical lines ar perpendicular

Examples: $y = -2x + 3$ OR $x = 7$ OR $y = 2/5x + 3$
$y = 1/2x + 8$ $y = 3$ $y = -5/2x + 5$

To determine if two lines intersect, are parallel, are perpendicular or are the same line:

1. convert them to slope-intercept form
2. compare their "m" values (slopes)
3. compare their "b" values (y-intercepts)
4. decide what relationship exists between the two lines:

- if both the slope and y-intercepts are the same they are the same line
- if only their slopes are the same they are parallel
- if their slopes are different they intersect in a non-specific way
- if their slopes are different and their product is -1, they intersect and are perpendicular
- if their y-intercepts only are the same, they intersect at the y-intercept

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Determine whether each pair of lines are parallel, perpendicular, the same line, or intersect in a non-specific way:

- $2x + 6y = 12$ AND $y - 2/3x = 1$

$-2x \quad -2x$

$\frac{6y}{6} = \frac{-2x}{6} + \frac{12}{6}$

$y = -\frac{1}{3}x + 2$

$m = -\frac{1}{3}$

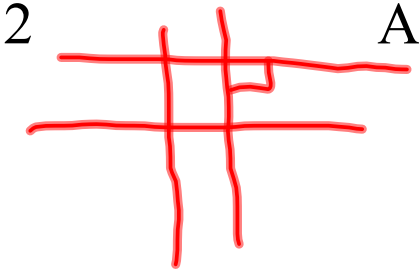
$+2/3x \quad +2/3x$

$y = \frac{2}{3}x + 1$

$m = 2/3$

intersect

- $x = 2$ AND $y = 9$



perp.

- $3x - 5y = 15$ AND $4y + 6x = 12$

$-3x \quad -5y = -3x + 15$

$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{15}{-5}$

$y = \frac{3}{5}x - 3$

$-6x \quad 4y = 6x + 12$

$\frac{4y}{4} = \frac{6x}{4} + \frac{12}{4}$

$y = \frac{6}{4}x + 3$

INTERSECT

- $2x - 4y = 14$ AND $-6x + 12y = -42$

Write an equation for the line that contains (5,1) and is parallel to y = 3/5x - 4

$y - 1 = \frac{3}{5}(x - 5)$

Write an equation for the line that contains (5,1) and is perpendicular to y = 3/5x - 4

$\frac{3}{5} \cdot -5 = ?$

$-3 \neq -1$

$y - 1 = \frac{-5}{3}(x - 5)$

$m = 3/5$

$m = -5/3$

Find an equation for a line that contains the point (0,-2) and is parallel to 3x - 5y = -15

$-3x \quad -3x$

$\frac{-5y}{-5} = \frac{-3x - 15}{-5}$

$y = \frac{3}{5}x + 3$

m

$y = \frac{3}{5}x - 2$

$y + 2 = \frac{3}{5}(x)$

Find an equation for a line that contains the point (-10,6) and is perpendicular 4x + 3y = 24

$\frac{-4x}{3} + \frac{24}{3}$

$m = -\frac{4}{3}$

$\frac{1}{m} = \frac{3}{4}$

$y - 6 = \frac{3}{4}(x + 10)$

$y - 6 = \frac{3}{4}x + \frac{15}{2}$

$+6 \qquad +6$

$y = \frac{3}{4}x + \frac{27}{2}$

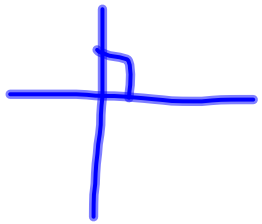
Find an equation for a line that contains the point (-4,-8) and is parallel to y = 3

$m = 0$

$y = -8$

$y = 0x - 8$

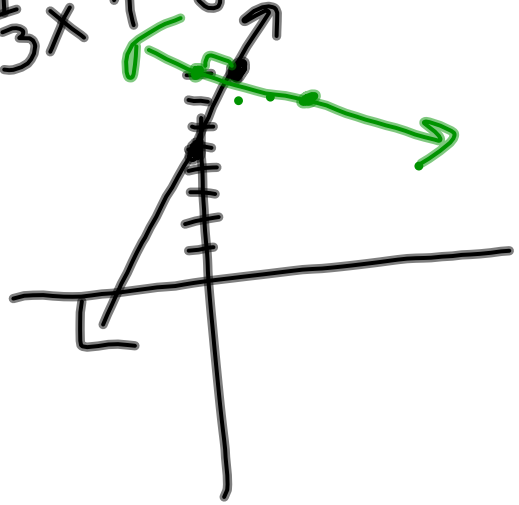
Find an equation for a line that contains the point (5,9) and is perpendicular to y = 7



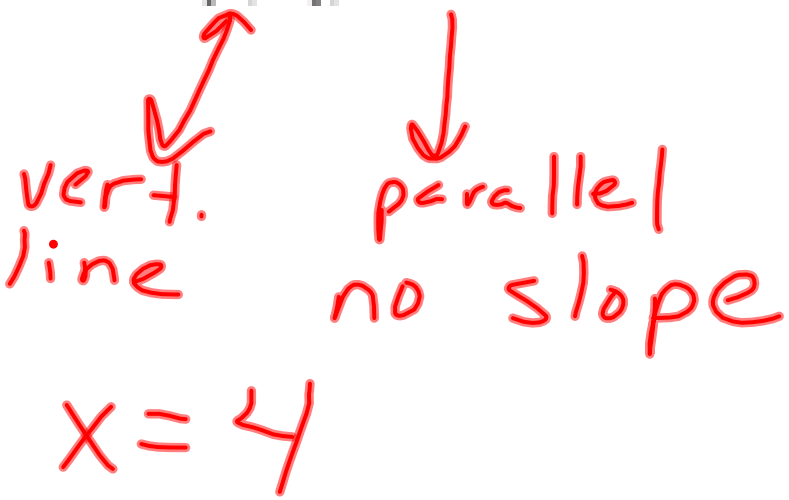
$x = 5$

$y = 3x + 5$
$y = 3x - 7$
} parallel

$y = 3x + 5$
$y = -\frac{1}{3}x + 8$
} Intersect &
$3 \cdot -\frac{1}{3} = -1$
Perpendic.



12. $(4, 2); x = -3$



$x = 4$

18. $y = 4x - 2$ ← $m = 4$

$-x + 4y = 0$ ← $m = \frac{1}{4}$

$+x \quad +x$

$\frac{4y}{4} = \frac{x}{4}$

$y = \frac{1}{4}x$

Intersect (neither)

8. $(2, -2)$; $y = -x - 2$

SI parallel to

$y = mx + b$ $m = -1$

$y + 2 = -1(x - 2) - 2$

$-2 \quad -2$

$y = -x + 2 - 2$

$y = -x$

20. $(-2, 3); y = \frac{1}{2}x - 1$

thru $(-2,3)$ $\perp$ m $y = -2$

$y - 3 = -2(x + 2)$

$y - 3 = -2x - 4$

$+3 \qquad +3$

$y = -2x - 1$

perp slopes have a prod of -1

24. $(1, -6)$; $x - 2y = 4$

$-x - x$

$-2y = -1x + 4$

$\frac{-2y}{-2} = \frac{-1x}{-2} + \frac{4}{-2}$

$y = \frac{1}{2}x - 2$

$m = \frac{1}{2}$

$\perp m = -\frac{2}{1}$

$\rightarrow$ SI

$y = mx + b$ PS

$y + 6 = -2(x - 1)$

$y + 6 = -2x + 2$

$-6 \quad -6$

$y = -2x - 4$ SI

16. $y - 4 = 3(x + 2)$ ← $m = 3$

$2x + 6y = 10$ ← $m = -1/3$

$-2x \quad -2x$

$\frac{6y}{6} = \frac{-2x + 10}{6} \quad \frac{}{6}$

$y = -\frac{1}{3}x + \frac{5}{3}$

Perpendicular