Chapter 4

An Introduction to Functions

The Big Ideas:

Functions

How can you represent and describe functions?

Students will represent functions using tables, equations, and graphs

Students will use function notation

Modeling

Can functions describe real-world situations?

Graphs will be used to relate two quantities

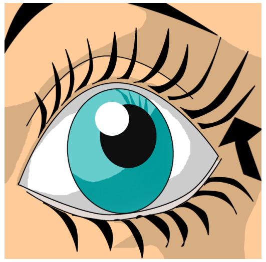
Students will model real-world situations that are continuous and those that are discrete

Essential Understandings

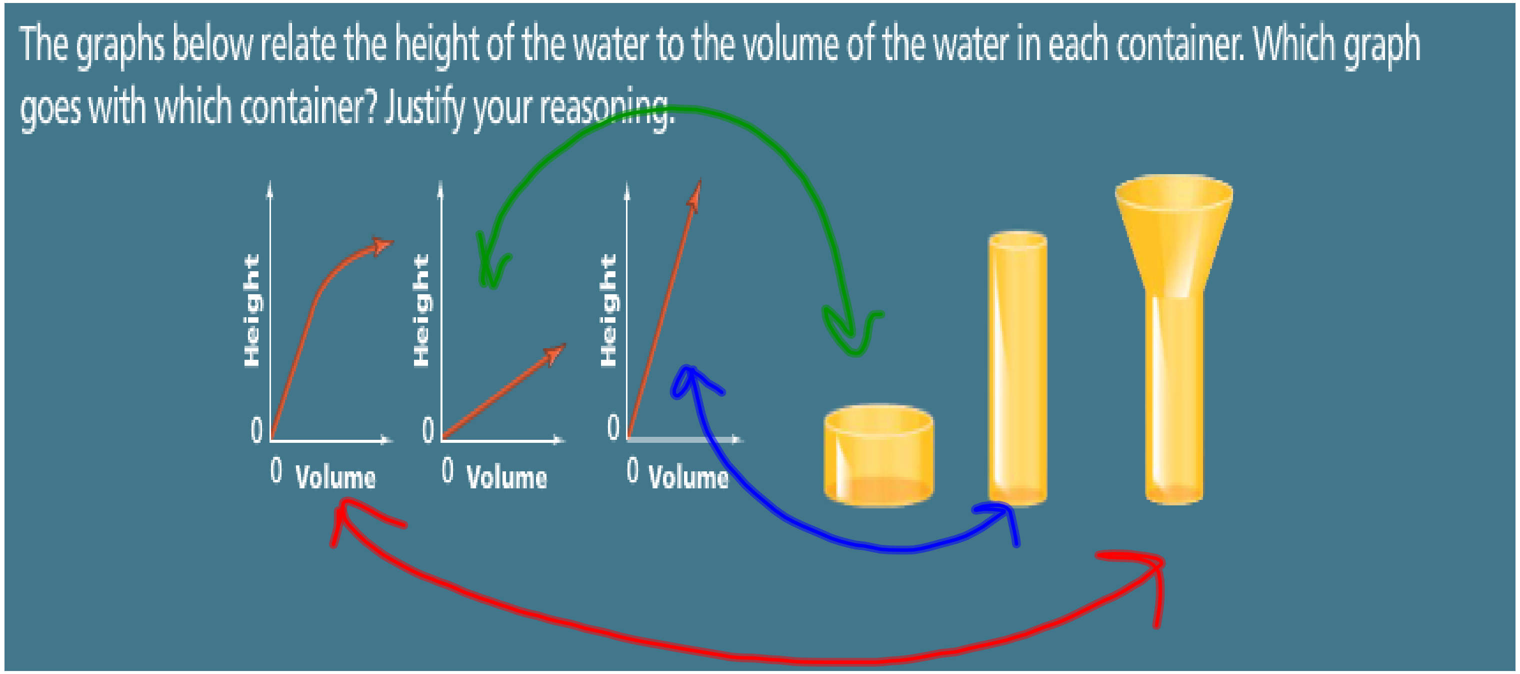
4-1: Using Graphs to Relate Two Quantities

Graphs can be used to visually represent the relationship between two variable quantities as they change

Tables and graphs can both show relationships between variables

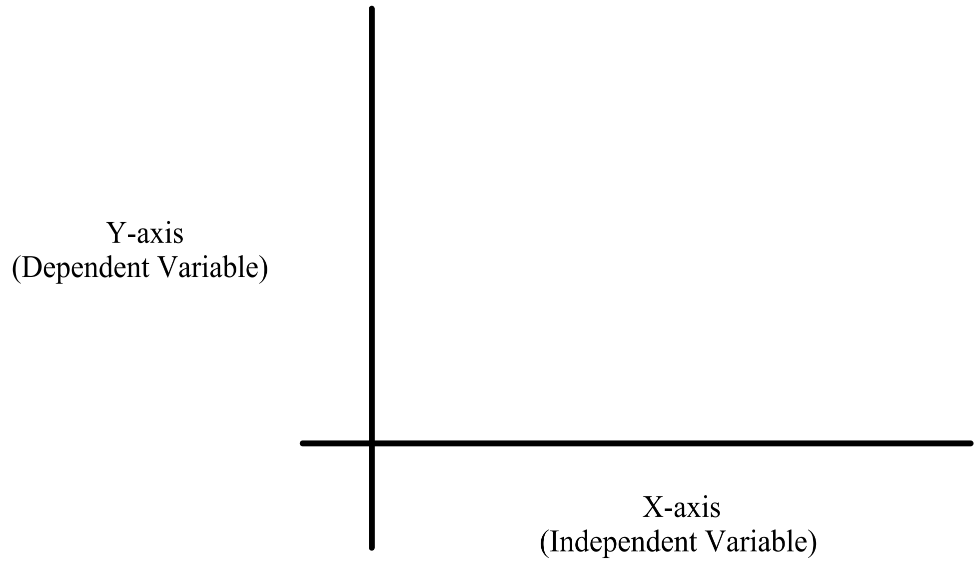


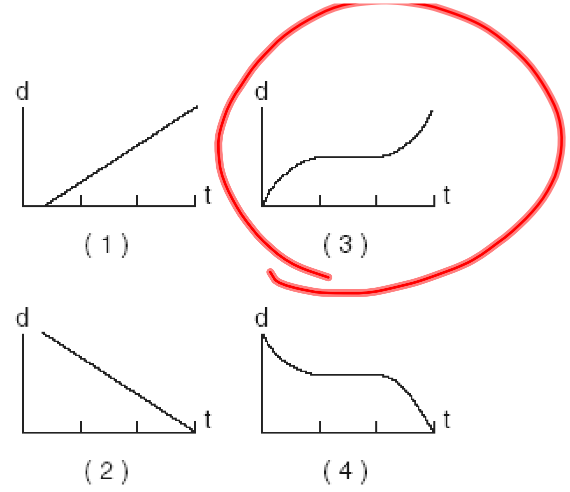
Eye Opener


The graphs below relate the height of the water to the volume of the water in each container. Which graph goes with which container? Justify your reasoning.
Height
0
0 Volume
Height
0
0 Volume
Height
0
0 Volume

Relating Graphs to Events

- **Graphs** are used to visualize relationships between two variables.
- The **independent variable** is measured on the x-axis (horizontal)
- The **dependent variable** is measured on the y-axis (vertical)
- The **domain** is the set of possible of x-values in the graph
- The **range** is the set of possible of y-values in the graph
- The intervals are the values indicated by the tick marks on the x & y-axis



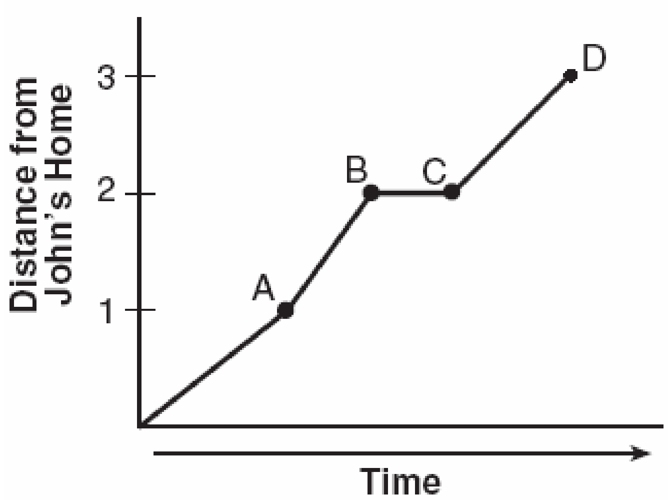


A bug travels up a tree, from the ground, over a 30-second interval. It travels fast at first and then slows down. It stops for 10 seconds, then proceeds slowly, speeding up as it goes. Which sketch best illustrates the bug's distance (d) from the ground over the 30-second interval (t)?

x = time in secs.
y = distance up tree

Move Me

Move Me



1. John left his home and walked 3 blocks to his school, as shown in the accompanying graph. What is one possible interpretation of the section of the graph from point *B* to point *C*?

[A] John arrived at school and stayed throughout the day.
[B] John returned home to get his mathematics homework.
[C] John reached the top of a hill and began walking on level ground.
[D] John waited before crossing a busy street.

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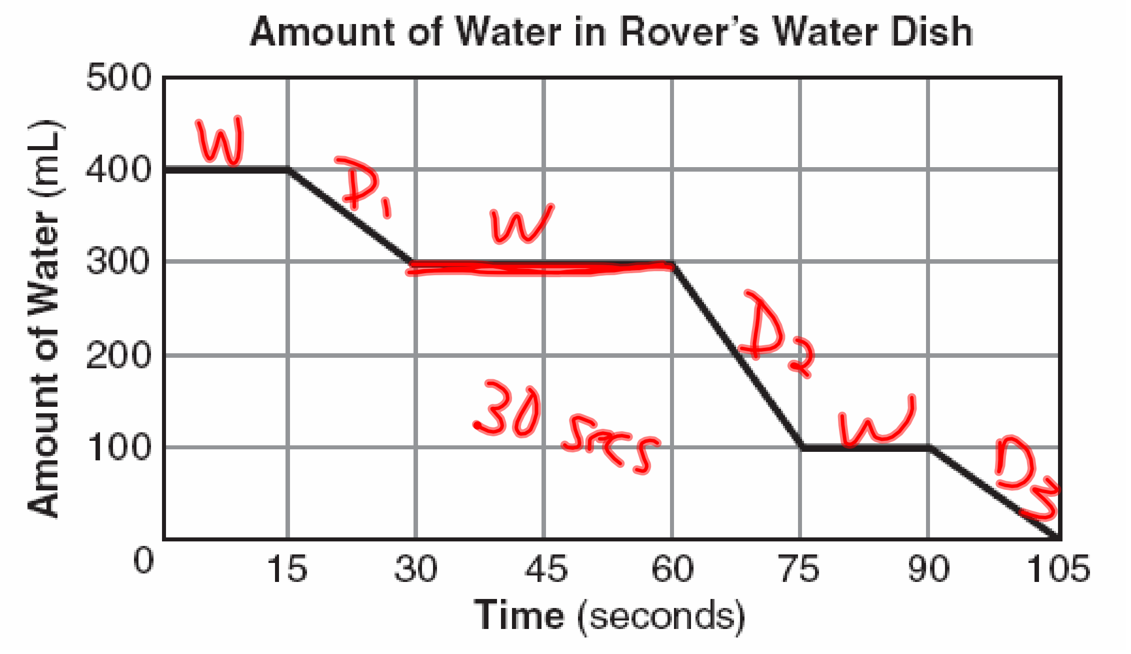
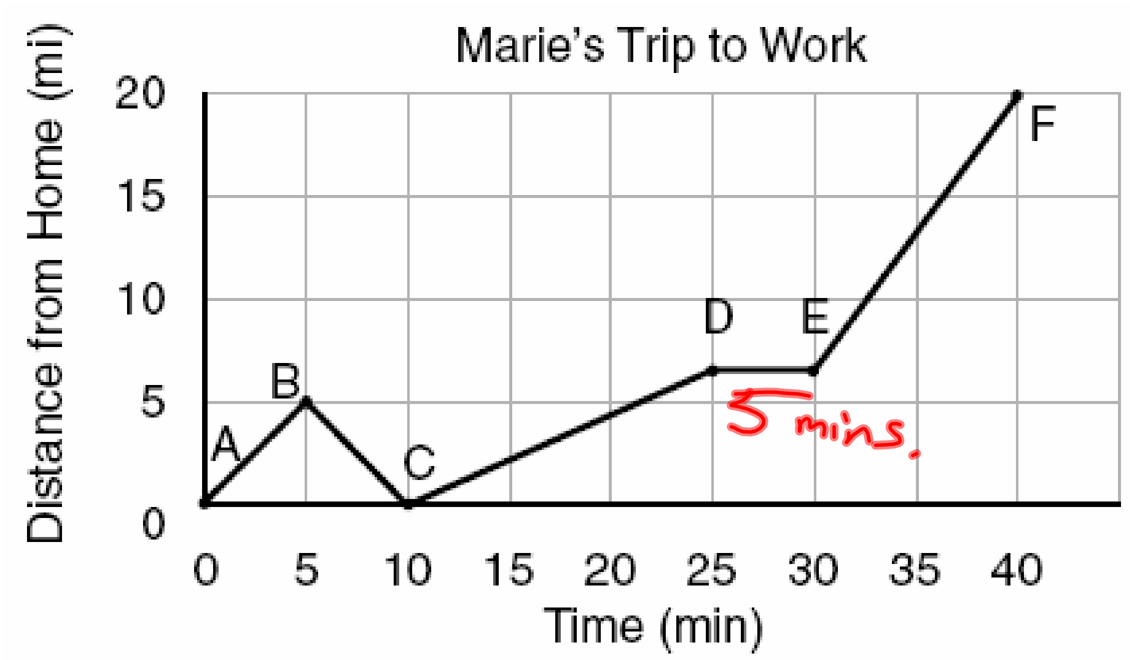
2. The accompanying graph shows the amount of water left in Rover's water dish over a period of time.

How long did Rover wait from the end of his first drink to the start of his second drink of water?





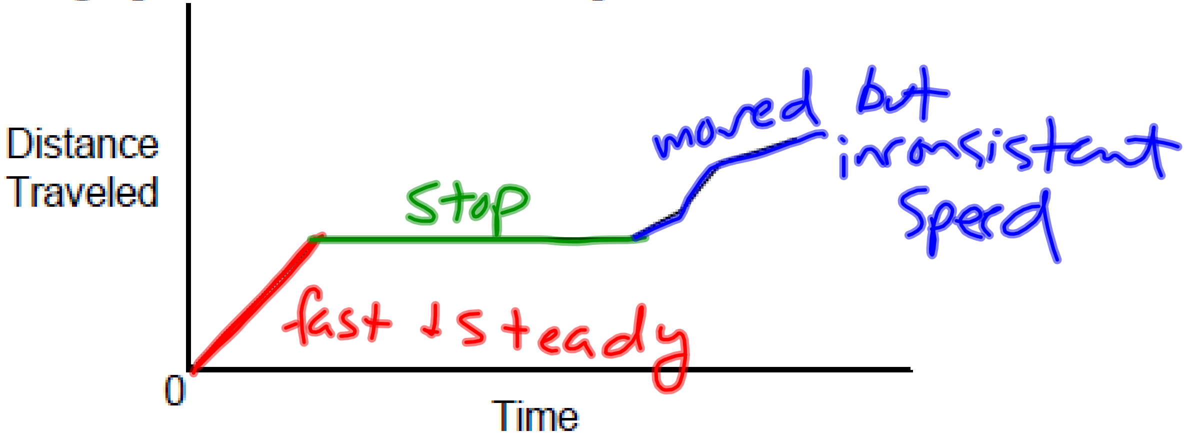
3. The accompanying graph shows Marie's distance from home (*A*) to work (*F*) at various times during her drive.

a Marie left her briefcase at home and had to return to get it. State which point represents when she turned back around to go home and explain how you arrived at that conclusion. BC

b Marie also had to wait at the railroad tracks for a train to pass. How long did she wait? DE

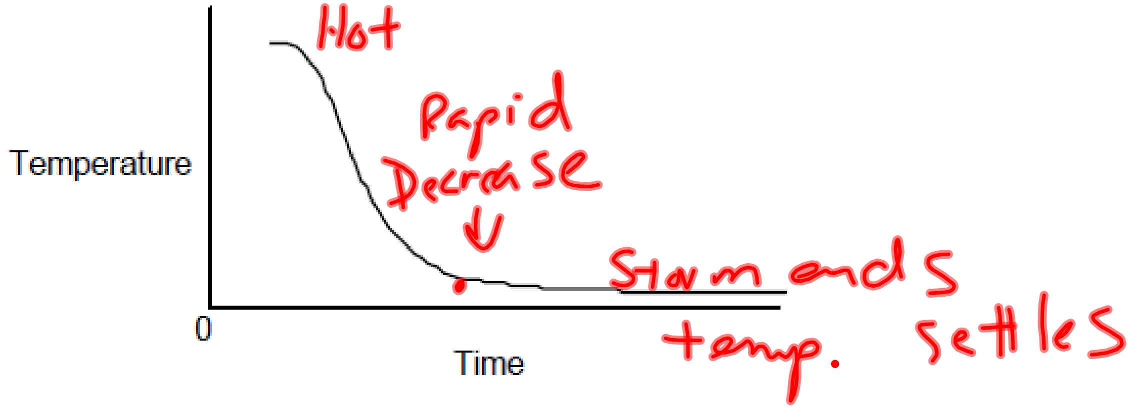
5 mins

6. The graph shows the relationship between time and distance traveled.

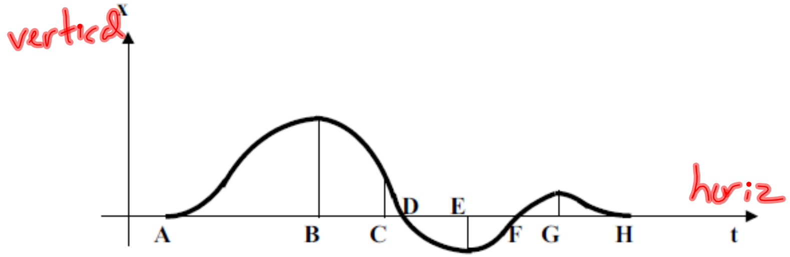


Describe a situation that the graph might show.

7. The graph shows the temperature during a recent thunderstorm. Describe what happened to the temperature during the storm.



Story #2: Side View of a Roller Coaster



Here the graph is being interpreted as the side view of a roller-coaster. As in the first story, t represents horizontal distance and x represents vertical distance.

1. **Where would you get the best view?** B
2. **Where is the safest place to stop the roller coaster car without putting on the brakes?**
3. **At what point might the roller coaster be going the fastest?** B → C
4. **What point is the scariest?**

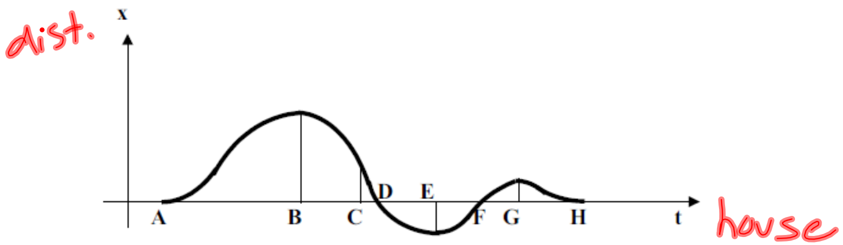
Answers:

Question 1: Where would you get the best view? Probably your answer would be at B, the highest point. Notice that although this point is characterised by height — the highest point — it is also one of the "flat" points, where momentarily the graph is neither going up or down.

Question 2: Where is the safest place to stop the roller-coaster car without putting on the brakes. This could be any of the points where the curve is flat (zero slope): A , B, E, G, H. But certain of these points would be a little dangerous. You could park the roller-coaster car precariously at B but a puff of wind might push it over the edge. Probably the safest place is at E.

Question 3: At what point might the roller-coaster car be going fastest? There are a lot of assumptions to be made here but you could argue that while it is going down-hill it picking up speed so that it might be going fastest at E. This is one of the minimum points (where the slope is zero). It might be going fast at the minimum at H but it hasn't been going down-hill for as long as when it hits B. Again it doesn't matter if you disagree with this answer as long as you can come up with good reasons.

Story #3: Position of a Cyclist



Both of the previous stories have had the graph give a literal picture of the situation and the interpretation has been relatively easy. But in this next story the horizontal axis represents time. The vertical axis represents distance along a north-south straight road. The graph gives the position of a cyclist as he rides up and down this perfectly straight, perfectly flat road. The graph curves around but not the cyclist.

The horizontal axis represents his house. Distances above this axis represent the cyclist being north of home and distances below this axis represent distances south. He leaves home at time A, riding north. Then at time B he slows down and stops in order to turn around. Between B and E he is going south, and at some point he passes his house. At time E he turns north again, and so on.

1. **How many times does ride "past his house?** D & F
2. **At what point is he riding the fastest?**
3. **Does he spend more time north of home or south of home?** north
4. **At what time did he realize it was dinner time?** # G

Answers:

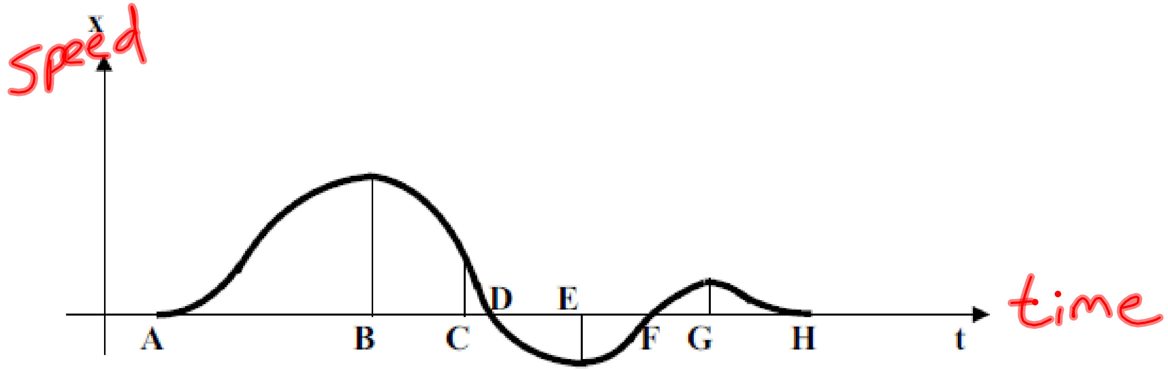
Question 1: He starts outside his house at time A and ends up outside his house again at time H. How many times does he ride right past his house? The answer is 2. He is riding past his house at times D and F.

Question 2: At what point is he riding fastest? The slope on this graph measures how fast he is travelling. The steepest point on the graph is at C and this represents the point where he is covering the most distance in each time interval.

Question 3: Does he spend more time north of home or south of home? Although we don't have a scale it is clear that he has spent more time north of home. He is north of home between time A and D and again between F and H. This clearly represents much more than half of the total time.

Question 4: At what point in time did he realise that it was dinner time? One can't be certain but a good guess would be at time G when he stopped and returned home to stay.

Story #4: Speed of a Train



In this story a train is moving along a straight, perfectly flat, piece of track going north-south. The horizontal axis represents time while the vertical axis represents the speed of the train in a northerly direction. The higher the graph, the faster the train is going. The horizontal axis represents a speed of zero, that is, the fact that the train is stationary. When the graph goes below the axis the train is going south. A speed of –60 kph is interpreted as a speed of 60 kph southwards.

1. **When is the train going fastest?** B
2. **When does the train change direction?** ~~B, E, G~~ D, F
3. **When is the train the furthest from the start?** H

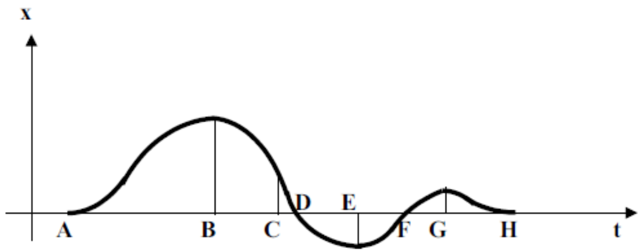
Answers:

Question 1: When is it going fastest?
Clearly at time B. The fastest it ever goes backwards is clearly at time E but since the distance to the axis is less than at B the speed it attains is less. Also at G it is going faster than just before or just after G, but clearly this isn't as fast as at time B. Notice that all three of these moments, B, E and G correspond to places where the slope of the graph is zero.

Question 2: When does it change direction?
The moments are D and F. Notice that at these times the graph cuts the axis, meaning that the train is momentarily stationary. It has to stop in order to change direction. The train is stopped at times A and H as well but because we don't know what happened before A or after H we can't say whether the train changes direction at these times.

Question 3: When is the train furthest from the start?
The answer is D. But follow this analysis carefully. From A to D the train is travelling north so at time D it is further north than at any time before. But it goes back down the line a certain distance and then back north again. If, by time H it hasn't returned to where it was at time D then it will be furthest from the start at time D. But if, after backing down the line, it goes north and passes the point where it was at time D then the time when it is furthest from the start is at

Story #5: Weekly Savings

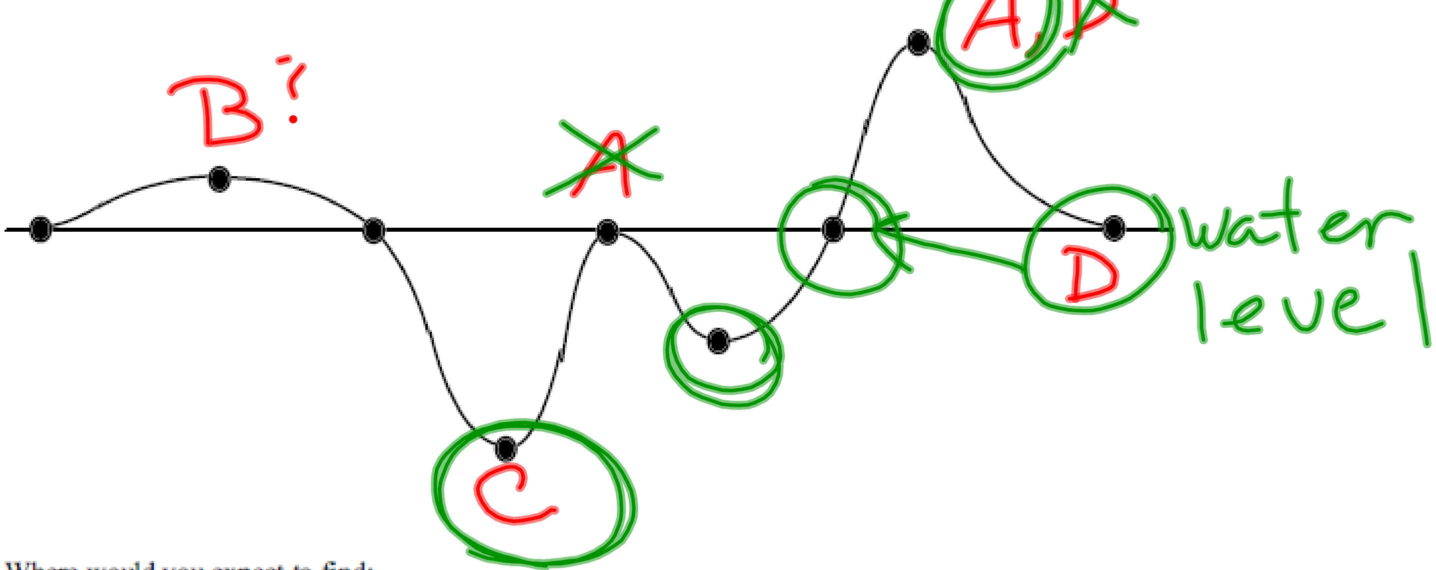


Once again the horizontal axis represents time, over the lifetime of Old Jock. Time A was when he started working and time H was when he died. The vertical axis represents his weekly savings. When the graph goes below the axis this represents negative savings, that is the times when he has to withdraw money from his account in order to live.

1. **During what period does he have to dip into his savings?**
2. **When does he have the largest bank balance?**
3. **When is his income exactly equal to his expenditures?**
4. **When does he get married?**

Answers:

Exercise 5: The following graph represents the Rhine River in the times of the Valkyries. The horizontal line represents the water level in the river.



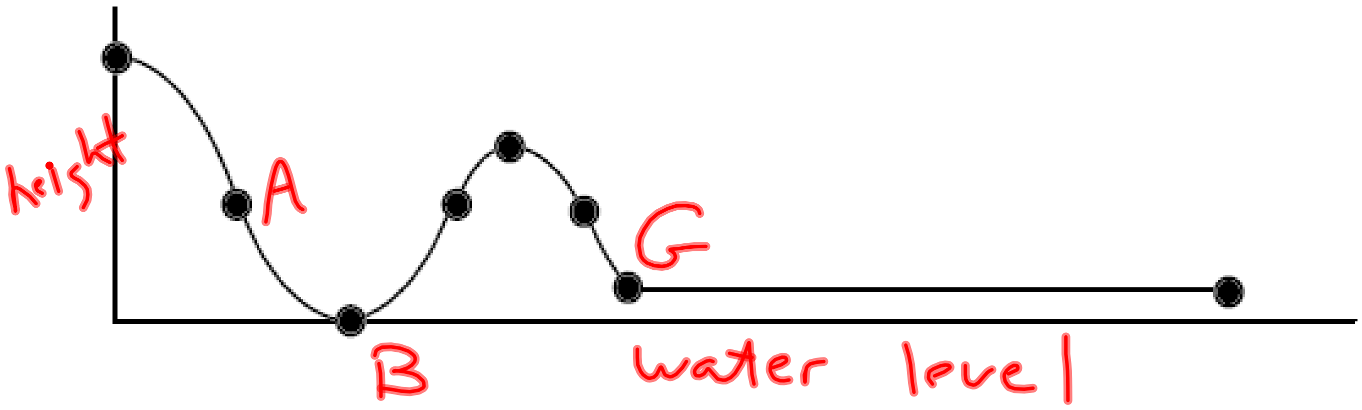
Where would you expect to find:
(a) Valhalla, Wotan's castle?
(b) The Lorelei who lure sailors onto the rocks.
(c) The Ring of the Nibelungen, in the deepest part of the Rhine.
(d) The army of the giant Fafner, assembled ready to make an assault on Valhalla.

Answers:

Exercise 5:

(a) H, on top of the highest hill allowing the castle to be easily defended.
(b) E, where the rocks come up to the surface but cannot be seen from above ground.
(c) D, a minimum on the curve, which represents the deepest part of the river.
(d) Possibly B, because flat solid ground would make a good assembly point, however the army would still have to cross the Rhine River, so perhaps I would be more appropriate for an immediate assault.

Exercise 6: The following graph represents the height above the water level of a bungy jumper who jumps off a bridge. The horizontal level represents water level.

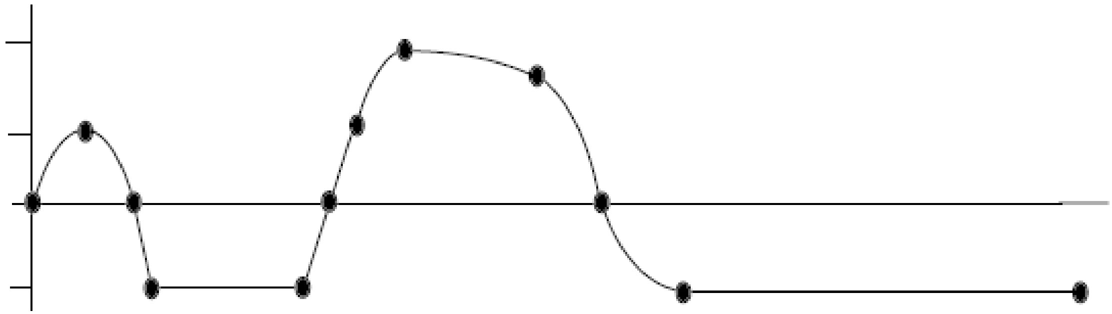


(a) Where is he falling fastest?
(b) When does he get his feet wet?
(c) Does he go under water at any stage? NO
(d) What might have happened at G?

hanging

Answers:

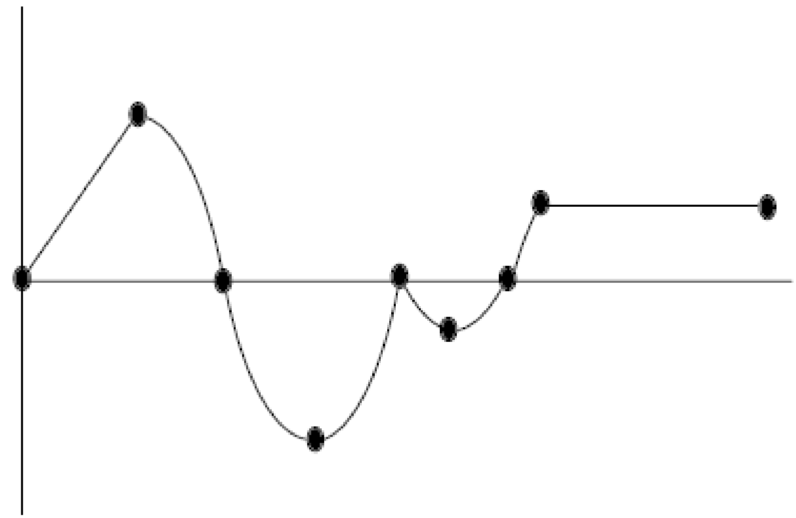
Exercise 8: The following graph represents the net inflow/outflow per day, into or out of a dam. The flow rate is plotted against time.



(b) When did it rain?
(c) When did it rain the hardest? (Ignore any delay there might be in the water reaching the dam.)
(d) When did the drought begin?
(e) Is the level of the dam at time F the same, lower or higher then at time A?
(f) When was the level of the dam the same as at time A?

Answers:

Exercise 7: In a laboratory experiment, the following graph represents the angle of a pendulum, measured from vertical, against time, t.

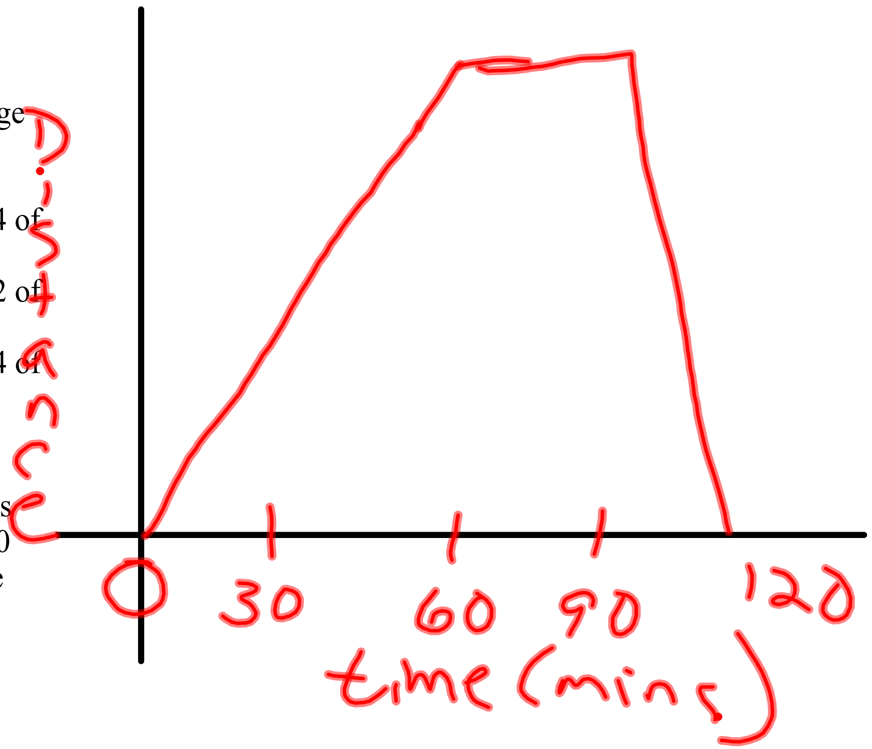


(a) What is happening between A and B?
(b) Where is the pendulum travelling fastest to the right?
(c) What might be happening at E?
(d) What might be happening between H and I?

Answers:

Problem 1: Hansel and Gretel

Hansel and Gretel leave their house and start walking towards their grandmother's house.
Once Hansel and Gretel start walking, they do not change their speed.
They can stop, though.
After 15 minutes of walking, Hansel and Gretel were 1/4 of the way to grandmother's house.
After 30 minutes of walking, Hansel and Gretel were 1/2 of the way to grandmother's house.
After 45 minutes of walking, Hansel and Gretel were 3/4 of the way to grandmother's house.
Hansel and Gretel reached grandmother's house after exactly one hour of walking.
When Hansel and Gretel had been at their grandmother's house for 30 minutes, they realized that they had only 30 minutes to walk back home in time for dinner, and leave grandmother's house to start walking back home.
They arrived home exactly thirty minutes after they left grandmother's.



On the coordinate plane, construct a graph of Hansel and Gretel's journey labeling distance from home as the dependent variable and time as the independent variable in the boxes provided.

Did they walk faster on the way to grandmother's house or on the way back from grandmother's house?

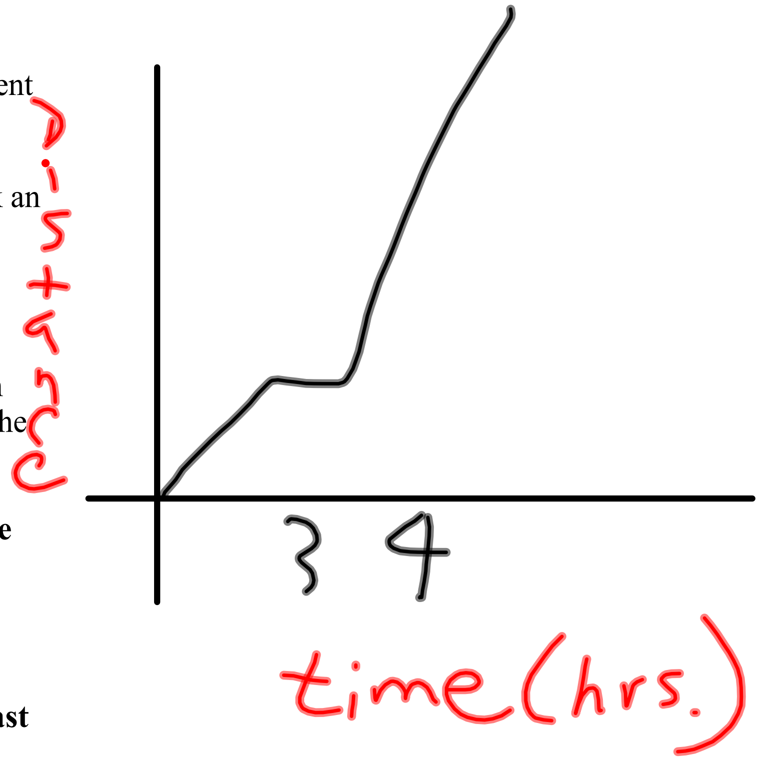
How is this illustrated on your graph?

Problem 3: Marathon Runner

Your friend ran in a marathon last weekend. Using the following information, construct a graph of her run with distance as the dependent variable and time as the independent variable.

- In the first two hours, she ran at a constant speed.
- In the third hour, however, your friend was tired and took an hour break.
- In the fourth hour, she ran twice as fast as in the first two hours to make up for the time she lost during her break.

On the coordinate plane, construct a graph of the marathon run labeling distance as the dependent variable and time as the independent variable in the boxes provided.



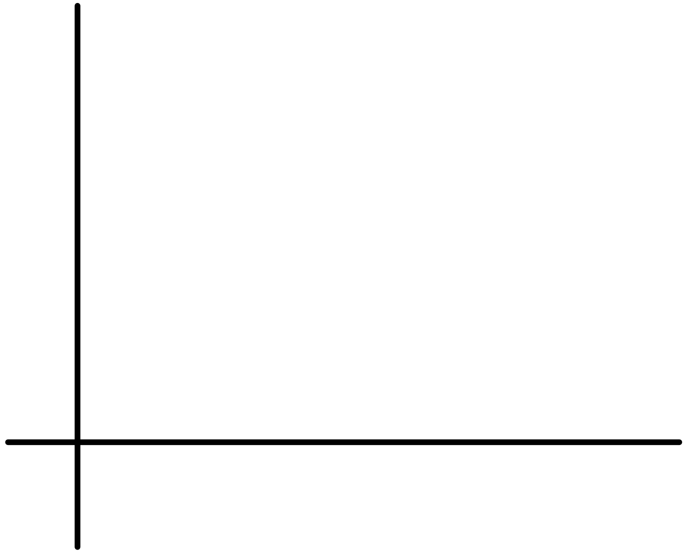
How far did your friend run in the first two hours of the race?

How far did she run in the fourth hour of the race?

How does your graph illustrate that she ran twice as fast in the fourth hour as in the first two hours?

Problem 4: Race Car Driver

Your uncle is a race car driver. One day you decide to go watch a race. Your uncle began the race well but got a flat tire 2.5 minutes into it and had to stop. You collected the following data while you were watching the race:



Time	Distance
0	0
0.5	0.5
1	1
1.5	1.75
2	3
2.5	4.5
3	4.5
3.5	4.5
4	4.5
4.5	4.5
5	4.5

Construct a graph of your uncle's race with distance as the dependent variable and time as the independent variable, labeling the axis appropriately in the boxes provided..

What was happening to your uncle's speed (not distance) between t=0 and t=2.5?

Between t=2.5 and t=5?

How does your graph illustrate these changes in speed?

Problem 5: Trip to School

Andrew rides his bike to school every day. He is usually not fully awake by the time he leaves, so it takes him time to gain his speed. After about 5 minutes he reaches the stop sign at the end of his road, and has to pause for a minute. He is going to get to school early so he just peddles along for a couple of moments. Suddenly his friend Dietrich comes up behind him and they start racing. They are both going pretty fast, but then Andrew remembers he left his science project on the table at home. Andrew quickly turns around and speeds back home to get it. Now he has only minutes until the start of school, and is going to be late if he doesn't hurry. Andrew pedals as fast as he can all the way back to school and makes it to class just in time for the first bell.

Construct a graph of this data. Label your axis appropriately in the boxes provided.

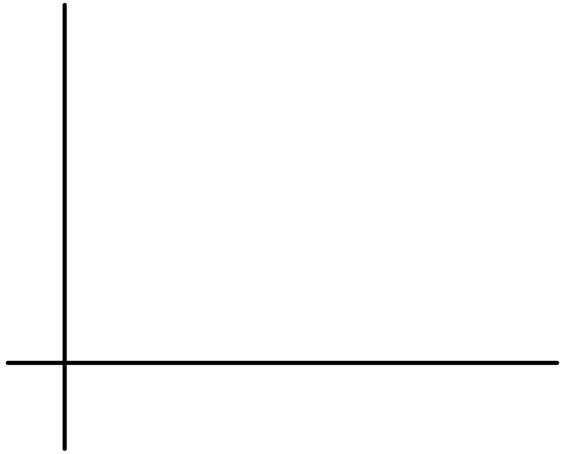
Think of two questions you could ask about this story and its graph and then answer them:

Question 1:

Question 2:

Problem 6: Foxes and Rabbits

Jason is an amateur naturalist and is counting the number of foxes and rabbits he sees in the park near his house. On his first time out he finds 200 rabbits but only 30 foxes. When he goes out to count the next year he finds that the foxes have eaten 75 rabbits and the fox population has increased to 50. The next year he counts only 25 foxes and the rabbit population is down to 50. Jason is concerned that something is happening to the rabbits and foxes. When he goes out the next year he finds there are 150 rabbits but the foxes are down to just 10. In his fifth year of counting he finds 200 rabbits and 30 foxes just like in the first year. Jason starts to see that as the number of rabbits goes up the foxes eat more. That means the number of rabbits goes down and the foxes goes up. But as the rabbits go down farther the foxes start to go down as well. With fewer foxes now the rabbit population can start to go up and start the cycle again.



Construct a graph of this data. Label your axis appropriately in the boxes provided.

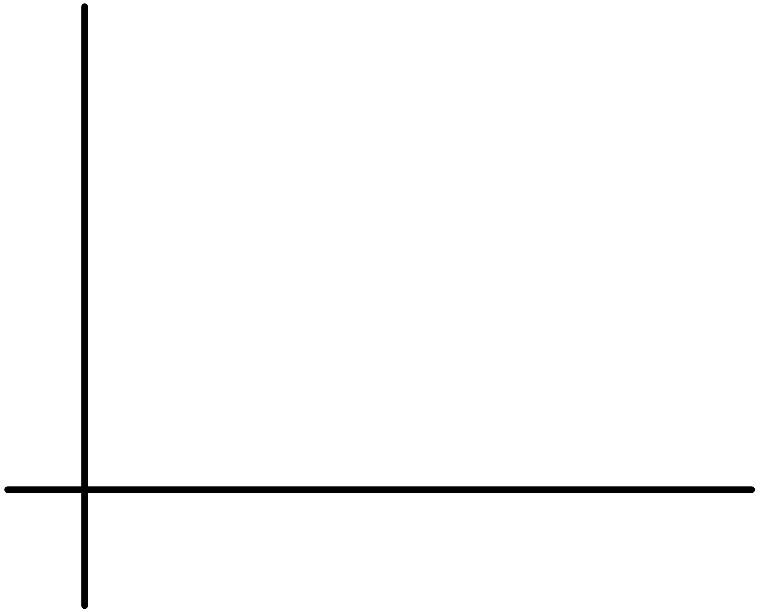
Think of two questions you could ask about this story and its graph and then answer them:

Question 1:

Question 2:

Problem 7: The Bakery Business

Leonard runs a bakery and diner in the middle of the town of Gumbin. He arrives at 5 am every day and opens the store. A few of the usual customers were in by 6:30 but it was relatively slow today. Most of the breakfast crowd arrived around 7 o'clock and the diner was full by 7:30. Most of the people headed off to work at 8, but some stayed a little longer. Around nine a group of tourists came in for a late breakfast. Business slowed down until 11 when people started pouring in for lunch. The diner was full until a little after 1:00 when the late lunch people left. Leonard was distressed by the lack of business between 1:30 and 4:00. Leonard's diner, named the Jang Express, was well known for its takeout also. The bakery and takeout boosted business as the workday ended from 4:00 to 5:00. Leonard and Ben, his cook, prepared for the steady rush of people that come to dinner from 6 to 8. The diner slowed down as it neared the closing time of 10:00pm.



Construct a graph of this data. Label your axis appropriately in the boxes provided.

Think of two questions you could ask about this story and its graph and then answer them:

Question 1:

Question 2: