Practice with 1-Variable Word Problems

Name:

Translating Word Problems into Symbols

Addition	increased by more than combined, together total of sum added to
Subtraction	decreased by minus, less difference between/of less than, fewer than
Multiplication	of times, multiplied by product of increased/decreased by a factor of (this type can involve both addition or subtraction *and* multiplication!)
Division	per, a out of ratio of, quotient of percent (divide by 100)
Equals	is, are, was, were, will be gives, yields sold for

Some Types of Word Problems

1. Distance Problems

   Formula:

   Rate (Speed) x Time = Distance

   - Types of Motion
     - Two vehicles moving away from each other from a common point – usually add their individual distances to get the total distance apart after a specified length of time
     - Two vehicles moving toward a common point – usually add their individual distances to get the total distance they were apart at the beginning
     - Two vehicles moving in the same direction arriving at a common destination– usually set their individual distances equal to each other
     - Two vehicles moving in any direction and not leaving from and / or arriving at a common destination – usually involves subtracting their individual distances
     - Round trips – going to and from the same end points – usually set the distance coming equal to the distance going

Sample 1.1: **Plane A leaves Los Angeles for New York @ 500 mph, at the same time plane B leaves New York for Los Angeles @650. If the distance from New York to Los Angeles is 3000 miles, how long will it take them to meet?**

Sample 1.2: **A freight train leaves a station traveling at 30 mph A passenger train leaves 1 hour later traveling at 50 mph. At what time will the passenger train overtake the freight train?**

Sample 1.3: **The fastest moving insect is the large tropical cockroach. It scurries at speeds of up to 2.3 feet per second. How many miles can a roach travel in 1.5 hours?**

Sample 1.6: **An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 miles; the entire trip took three hours. Find the distance from the airport to the corporate offices.**

2. Solutions / Mixture Problems

   Formula:

   Volume of Solution x Strength of Solution = Amount of Pure Element in Solution

   - When pure element is added the strength is 100%
   - When a solution without any identified strength, or with no element present is added, it has a strength of 0% (like water)

Sample 2.1: **1 ounce of the mixture containing 6% salt is to be mixed with 2 ounces of a mixture which is 15% salt, in order to obtain a new solution of salt. What is the percentage of salt in the resulting solution?**

Sample 2.2: **1 ounce of the mixture containing an unknown percentage of salt is to be mixed with 2 ounces of a mixture which is 15% salt, in order to obtain a solution which is 12% salt. What was the percentage of salt in the first solution?**

Sample 2.3: **A mixture containing 6% salt is to be mixed with 2 ounces of a mixture which is 15% salt, in order to obtain a solution which is 12% salt. How much of the first solution must be used?**

Sample 2.4: **Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, you decide to mix 10% solution with 30% solution, to make your own 15% solution. You need 10 liters of the 15% acid solution. How many liters of 10% solution and 30% solution should you use?**

3. Generic word problems

   - **No Formulas apply**
   - Define your variable and write and solve the equation

Sample 3.1: **Gilda is looking at the skateboards in the window of Thunder Mountain Sport store. The Blue Thunder skateboard costs $8.47 more than the Silver Streak skateboard, but $12.95 less than the Wizard skateboard. The three skateboards together cost $160.00. How much does each skateboard cost?**

Sample 3.2: **A pole is standing vertically in a lake in such a way that 1/5 of the pole is in the mud, 2/3 is in the water, 1/8 is above the water, and a piece of the top measuring 1 ft. 3 ins. is broken off. What is the depth of the lake?**

4. Coins / Money

   Formula:

   Number of Coins x Individual Value of that Coin = Total Value of all Coins of that Type

   - Remember to be consistent – keep all values in dollars (decimals) or all in cents
     - To convert from dollars to cents multiply by 100
     - To convert from cents to dollars divide by 100

Sample 4.1: **Your uncle walks in, jingling the coins in his pocket. He grins at you and tells you that you can have the coins if you can figure out how many of each kind of coin he is carrying. You're not too interested until he tells you that he's been collecting those gold-tone dollar coins. The twenty-six coins in his pocket are dollars and quarters, and they add up to seventeen dollars. How many of each coin does he have?**

Sample 4.2: **A collection of 33 coins, consisting of nickels, dimes, and quarters, has a value of $3.30. If there are three times as many nickels as quarters, and one-half as many dimes as nickels, how many coins of each kind are there?**

Sample 4.3: **A wallet contains the same number of pennies, nickels, and dimes. The coins total $1.44. How many of each type of coin does the wallet contain?**

5. Combinations (Tickets, Mixed Nuts, Coffee Blends, etc.)

   Formula:

   Number of Items Sold x Individual Price of ea Item = Total Value/Cost of Items Sold

   - Remember to be consistent – keep all values in dollars or all values in cents

Sample 5.1: **Tyler's parents pay him $0.50 to do the laundry and $1.25 to mow the lawn. In one month, he does the laundry 6 more times than he mows the lawn. If his parents pay him $12.75 that month, how many times did he mow the lawn**?

Sample 5.2: **The total receipts for a basketball game are $1400 for 788 tickets sold. Adults paid $2.50 for admission and students paid $1.25. How many of each kind of tickets were sold?**

Sample 5.4: **The Madison Local High School marching band sold gift wrap to earn money for a band trip to Orlando, Florida .The gift wrap in solid colors sold for $4.00 per roll ,and the print gift wrap sold for $6.00 per roll .The total number of rolls sold was 480 ,and the total amount of money collected was $2340. How many rolls of each of each kind of gift wrap were sold?**

6. Interest

Formula:

Interest (Money Paid or Earned) = Principle (Amount Invested) x Rate (Percent of Return per Year) x Time (Number of Years)

- Don't forget to convert your percents into decimals (divide by 100)
- Remember to be consistent – if percentage rate is in years then time must also be in years
- Assume time to be one year unless told specifically otherwise

Sample 6.1: Mr. Warren Buffett invested $20000. Part of it he put in the bank earning 5% interest. Part of it he invested in bonds which pay a 7% return. How much money did he put in each, if his annual income from both these investments was $1200?

Sample 6.2: Manuel's savings account has a return of 7.5%. After one year, during which Manuel did not add to or withdraw from his account, the account had $1410.40 in it. What amount was in the account at the beginning of the year?

Sample 6.3: Adam Goldberg put half of his money into a bank and invested another half of it in bonds. The bank pays 5% interest. The bonds pay a 7% return. How much money did he invest in each, if his annual income from the bonds was $600 more than the interest from the bank?

7. Number / Consecutive Numbers Problems

Formulas:

- Consecutive Integers: x, x + 1, x + 2, x + 3, …
- Consecutive Even / Odd Integers: x, x + 2, x + 4, x + 6, …
- Consecutive Multiples of 3, 4, 5, "y" : x, x + y, x + 2y, x + 3y, … (where y = "the multiple"

Sample 7.1: The sum of three consecutive numbers is 18. What are these numbers?

Sample 7.2: The sum of two numbers is 5 times their difference. If one exceeds the other by 7, what are the numbers?

Sample 7.3: Find three consecutive even integers so that the largest is 2 times more than the smallest.

Sample 7.4: The sum of two numbers is 41. The larger number is 1 more than 4 times the smaller number. What are these numbers?

8. Geometry

Formulas:

- Perimeter = Sum of the lengths of all sides
  - Square of side length "S" = 4S
  - Rectangle of length "L" and width "W" = 2L + 2W
  - Equilateral geometric figure of side length "L" = Numbers of sides x L
- Complementary Angles = 2 angles whose sum is 90°
- Supplementary Angles = 2 angles whose sum is 180°
- The sum of the measures of the three angles in a triangle is 180°

Sample 8.1: In a right triangle, one of the acute angles is 2 times as large as the other acute angle. Find the measure of the two acute angles.

Sample 8.2: The length and the width of a rectangle are given by consecutive integers. The perimeter of the rectangle is 38 cm. Find the area of the rectangle.

Sample 8.3: One angle is twice the measure of the other. If the angles are supplementary, what are their individual measures?

9. No solution – word problems with an unacceptable solution or whose equation simplifies to the empty set

- Solutions which are unreasonable – i.e. negative ages, fractional coins

- Solutions which contradict information given in the problem – i.e. looking for consecutive even numbers and answer is consecutive odd numbers; looking for measures of angles in a triangle and one of the angles exceeds 180° or is negative, etc.

Sample 9.1: The sum of three consecutive even numbers is 45. What are the numbers?

Sample 9.2: Jake has nickels, dimes and quarters in his pocket totaling $2.45. He has twice as many quarters as dimes, and the same number of nickels and quarters. How many of each coin does he have?

10. Age – 1 or 2 variables
    - Formula: Identify the time periods detailed in the word problem and create a separate column for each time frame (i.e. now, in 3 years, 2 years ago, etc.)
    - Be very careful that your chart is consistent from column to column – don't introduce data into your chart that contradicts the previous or next column; do not introduce data into your chart that should be used instead in writing your equation.

Sample 10.1: Bob's father is 3 times as old as Bob. 4 years ago, he was 4 times older. How old is Bob?

Sample 10.2: In January of the year 2000, my husband John was eleven times as old as my son William. In January of 2012, he will be three times as old as my son. How old was my son in January of 2000?

Sample 10.3: In three more years, Miguel's grandfather will be six times as old as Miguel was last year. When Miguel's present age is added to his grandfather's present age, the total is 68. How old is each one now?

11. Digit Problems – 2 variables
    - Remember place value is as important as the number: For the number "x"
        - Its value as a ones digit is x
        - Its value as a tens digit is 10x
        - Its value as a 100s digit is 100x and so on
    - In a two digit number where "x" is the tens digit and "y" is the ones digit the number is "10x + y" (i.e. 35 = 10 x 3 + 5; 85 = 10 x 8 + 5)
    - The reverse of the number above would be "10y + x" (the reverse of 35 is 53 = 10 x 5 + 3)
    - In a three digit number where "x" is the hundreds digit, "y" is the tens digit and "z" is the ones digit, the number would be "100x + 10y + z" (i.c. 357 = 100 x 3 + 10 x 5 + 7) and its reverse would be "100z + 10y + x" (i.e. 753 = 100 x 7 + 10 x 5 + 3). Note the place value of the tens digit remains the same originally and reversed.

Sample 11.1: What two-digit number is equal to twice the product of its digits?

Sample 11.2: Find a two-digit number such that if you reverse its digits to get another two-digit number, then add this to the original number, you get 143

Sample 11.3: The units digit of a two digit number is twice the ten's digit. If the digits of the number were reversed, the resulting number would be six less than twice the original number. Find the original number.

12. Work Problems – 1 variable
    - Formula: Rate x Time = Work

- Each person's work rate is the fractional amount of the job they can complete on their own in one hour: Rate = 1/time (i.e. 4 hrs. to do a job would mean a rate of ¼)
- The total work = "1" complete job whatever the job may be
- Usually time is the unknown.
- Usually add work (RT) done by one person alone to work done by other person alone (RT) to equal (1) job.

Sample 12.1: It takes John 5 hours to paint a house. It takes Bob 3 hours to paint the same house. How long will it take to paint the house if both of them work together?

Sample 12.2: Suppose one seamstress can sew the entire dress in twelve hours, and the second seamstress takes eight hours. How long would it take the two seamstresses together to sew the dress?

Sample 12.3: One pipe can fill a pool 1.25 times faster than a second pipe. When both pipes are opened, they fill the pool in five hours. How long would it take to fill the pool if only the slower pipe is used?